Performance response of wireless channels for quantitatively different loss and arrival statistics*

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Abstract

In this paper we propose a cross-layer performance evaluation framework for wireless channels and subsequently explore the performance response at the FEC and ARQ enabled data-link layer in terms of frame losses and delays for different first- and second-order error and arrival statistics. For a wireless channel model to be appropriate for various FEC capabilities without the need for extensive measurements of the frame error process for each particular FEC code, the error process of the wireless channel is modeled at the physical layer. We assume weak stationary property for bit error observations and model them using a hidden Markov model. The associated parameters matching algorithm allows to explicitly capture bit error rate and lag-1 autocorrelation of the bit error process. To explore the performance response of the wireless channel at the data-link layer a cross-layer extension

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of the bit error model to the data-link layer is then developed. The performance of applications is evaluated using the queuing-theoretic approach that allows both arrival and error processes to be autocorrelated and still retains analytical tractability. The proposed methodology allows to obtain estimators of frame loss and delay probabilities in presence of FEC and ARQ procedures at the data-link layer eliminating the need for time-consuming simulations and extensive measurements of wireless channel characteristics for different error correction capabilities of the data-link layer. It is analytical in nature, efficient for small and moderate frame sizes and suitable for performance control purposes where fixed size frames are used at the data-link layer. Particularly, it provides a way to choose the required correction capability of the FEC code resulting in best possible performance at the data-link layer. Numerical results indicate that first- and second-order bit error and frame arrival statistics significantly affect performance parameters provided by a wireless channel and should be taken into account when choosing an appropriate correction capability of the FEC code for given wireless channel conditions.

Keywords: cross-layer modeling, FEC optimization, queuing analysis.

1 Introduction

Wireless channels are characterized by highly dynamic time-varying nature. The propagation path between the transmitter and a receiver may vary from simple line-of-sight (LOS) to very complex ones. A major consequence of propagation characteristics is that the performance of wireless channels is heavily affected by the incorrect reception of channel symbols due to insufficient values of the signal-to-noise ratio (SNR) at some instants of time [2, 3]. Data-link layer error correction techniques such as forward error correction (FEC) and automatic repeat request (ARQ) may allow to recover from these errors locally. However, bit errors may still propagate to higher layers resulting in loss of protocols data units (PDU) at those layers. Satisfactory performance of applications at the data-link layer is then of paramount importance for overall quality of the service perceived by the end user.

ARQ techniques eliminate the influence of bit errors allowing to retransmit incorrectly received frames. To notify the sender about the erroneously received frame, ARQ protocols require a feedback channel. When wireless channel conditions are relatively 'bad' ARQ may introduce significant delays that are not always tolerable for delay-sensitive applications such as real-time two-way voice communication or streaming video. FEC procedures use proactive approach eliminating the influence of bit errors in advance, introducing error correction redundancy. This redundancy is efficiently exploited at the receiver to recover from bit errors. The major advantage of FEC techniques for delay-sensitive applications is that they do not introduce long delays allowing some information to be lost. Depending on a particular wireless access technology, FEC capabilities can be implemented at the physical or data-link layers. Due to different and complementary advantages, FEC and ARQ are often used in combination.

Recently, to study performance of FEC, ARQ and hybrid ARQ/FEC techniques wireless channel models at the data-link layer have been used (see [4, 5, 6] among others). These models are represented by the frame error process and *implicitly* include FEC capabilities of the data-link layer. As a result, such models are limited to a given FEC procedure and not efficient for performance optimization and control of wireless channel performance when, depending on the state of the wireless channel, different FEC codes are dynamically used. Indeed, for each particular FEC code a separate set of measurements is required to parameterize the wireless channel model at the data-link layer. On the other hand, bit error models may provide the required versatility of the modeling and optimization environment. However, these models cannot be directly used in performance evaluation studies at the data-link or higher layers and must be previously extended to the layer at which performance of applications is to be evaluated. For such an extension to be accurate, we have to take into account specific peculiarities of underlying layers including data-link error concealment techniques, segmentation procedures between adjacent layers, etc. An adequate wireless channel model for performance optimization and control purposes must be cross-layer complex function of the bit error process at the layer of interest. Therefore, modeling of the bit error process observed on wireless channels is an important issue providing a starting point in performance analysis, optimization and control of wireless channel performance.

In this paper, using the cross-layer performance evaluation approach we explore the performance response of the dedicated constant bit rate (CBR) wireless channel at the data-link in terms of probability functions of the number of lost frames and the delay of a frame. We assume that data are transmitted in fixed length frames and FEC and ARQ procedures are used at the data-link layer. For this configuration of the CBR contention-free wireless channel we develop an analytical performance evaluation framework. Using this framework, we show that the loss and delay performance experienced by applications at the data-link layer varies substantially for different error correction capabilities of the data-link layer and input statistics of the bit error and frame arrival processes.

This work has been partially inspired by excellent theoretical studies of S.-Q. Li and C.-L. Hwang [7], and B. Hajek and L. He [8] who considered performance response of the service process on a perfect (error-free) fixed link modeled by a queuing systems for different input statistics of the packet arrival process. In [7] authors concluded that the second-order statistics of the packet arrival process (autocorrelation function (ACF) or its power spectrum) significantly affect performance measures of applications running over the CBR error-free channel. Given the same assumptions of the service process, B. Hajek and L. He considered the performance response of a number of packet arrival processes with the same mean and ACF. They have shown that even for the same mean and ACF, the form of the probability distribution function of the number of arrivals may severely affect performance measures of the service process. In [9] authors considered performance of applications running over a wireless channel assuming that the bit error probability remains constant during the frame transmission time. Among other conclusions they have shown that both first- and second-order statistics of the frame error and frame arrival processes may affect performance measures of the service process of wireless channels. In [10] authors considered the performance of delay-sensitive applications running over CBR wireless channels for different error correction capabilities of FEC codes at the data-link layer. They have shown that the loss response of the wireless channel may vary substantially for different first- and second-order bit error statistics. We continue efforts of [7, 8, 9, 10] relaxing restrictive assumptions of [9, 10] and exploring how performance of the wireless channel varies in response to changes in bit error and frame arrival statistics and error correction capabilities of the data-link layer. Our numerical results indicate that the interplay between these properties is of paramount importance for optimal performance of applications running over wireless channels.

The rest of the paper is organized as follows. The frame arrival process is introduced in Section 2. The frame error model is developed in Section 3. In that section we consequently review the related work, consider the effect of error propagation to higher layers, formulate the bit error model and extend it to the data-link layer taking into account FEC capabilities. Then, the service model of the wireless channel is introduced in Section 4. This model is evaluated for performance parameters of interest in Section 5. Numerical results for a wide range of input statistics and FEC codes are presented and discussed in Section 6. Conclusions are drawn in the last section.

2 Arrival model

In this paper both arrival and error processes are assumed to be covariance stationary and modeled by special cases of discrete-time batch Markovian process (D-BMP). The latter is known as discrete-time batch Markovian arrival process (D-BMAP) in traffic modeling and hidden Markov chain (HMM) in signal processing. In this section, we briefly review probabilistic characteristics of D-BMP and define the frame arrival model.

2.1 Discrete-time batch Markovian process

Assume a discrete-time environment, i.e. time axis is slotted, the slot duration is constant and given by $\Delta t = (t_{i+1} - t_i)$, $i = 0, 1, \dots$. Consider the discrete-time homogenous ergodic Markov chain $\{S(n), n = 0, 1, \dots\}$ defined at the state space $S(n) \in \{1, 2, \ldots, M\}$. Let D be its transition probability matrix and $\vec{\pi} = (\pi_1, \pi_2, .., \pi_M)$ be the row array containing equilibrium state probabilities of this Markov chain. Let then $\{W(n), n = 0, 1, ..\}$ be D-BMP whose underlying Markov chain is $\{S(n), n = 0, 1, \ldots\}$. According to D-BMP, the value of the process is modulated by the discrete-time Markov chain $\{S(n), n = 0, 1, \ldots\}$, $S(n) \in \{1, 2, \ldots, M\}$. We define D-BMP as a sequence of matrices $D(k)$, $k = 0, 1, \ldots,$ each of which contains probabilities of transition from state to state with $k = 0, 1, \ldots,$ arrivals, respectively. For example, element $d_{ij}(0)$ defines transition from state i to state j without any arrivals, element $d_{ij}(k)$ defines transition from state i to state j with a batch arrival of size k. It is easy to see that for each pair of states $i, j \in \{1, 2, \ldots, M\}$ the following

$$d_{ij}(k) = Pr\{W(n) = k, S(n) = j | S(n-1) = i\}, \qquad k = 0, 1, \ldots, \quad (1)$$

are conditional probability functions of D-BMP.

Let the vector $\vec{G} = (G_1, G_2, \ldots, G_M)$ be the mean vector of D-BMP, where $G_i = \sum_{j=1}^{M} \sum_{k=1}^{\infty} k d_{ij}(k)$, $i = 1, 2, \ldots, M$. The mean process of D-BMP is denoted by $\{W_G(n), n = 0, 1, \ldots\}$ with $W_G(n) = G_i$, when the Markov chain is in the state i in the time slot n. The ACF of the mean process of D-BMP is [11]

$$R_G(i) = \sum_{l, l \neq 1} \phi_l \lambda_l^{i-1}, \qquad i = 1, 2, \ldots, \quad (2)$$

where $\phi_l = \vec{\pi} \sum_{k=0}^{\infty} k D(k) \vec{g}_l \vec{h}_l \sum_{k=0}^{\infty} k D(k) \vec{e}$, λ_l is the ls eigenvalue of D, $\vec{g}_l$ and $\vec{h}_l$ are ls left and right eigenvectors of D, respectively, and $\vec{e}$ is the vector of ones. Note that ACFs of D-BMP and its mean process are generally different [11].

The number of terms composing ACF of the mean process of D-BMP depends on the number of eigenvalues. The number of eigenvalues is the function of the number of states of the modulating Markov chain. Thus, varying the number of states of the modulating Markov chain we vary the number of terms composing the ACF. Recall that it is also allowed for D-BMP to have different probability functions for each different pair of states. These properties have been used in many studies to derive models of various traffic sources with sophisticated distributional and autocorrelational properties (see [12, 13, 14] among others).

Without affecting abovementioned autocorrelational properties we allow our D-BMP to have conditional probability functions that depend on the current state only. In this case, $D(k)$, $k = 0, 1, \dots$ have the same elements on each row. This process is known as Markov modulated batch process (MMBP). It is important that this process still has ACF distributed according to (2). In what follows, we also use only two states of the modulating Markov chain. For this reason, we refer to such processes as switched ones.

2.2 Frame arrival model

Let us denote the frame arrival process by $\{W_A(n), n = 0, 1, \dots\}$. In this paper, terms 'frame' and 'codeword' are used interchangeably assuming that a single frame consists of exactly one codeword. This requirement is not fundamental and can be relaxed when needed as explained below.

When MMBP $\{W_A(n), n = 0, 1, \dots\}$ is allowed to have only two states of the modulating Markov chain, $S_A(n) \in \{1, 2\}$, and each state is associated with Poissonally distributed number of arrivals in a single slot, it reduces to switched

Poisson process (SPP). The marginal distribution of SPP is a weighed sum of two Poisson distributions, where weighting coefficients are given by elements of the stationary distribution of the modulating Markov chain as follows

$$\pi_{1,A} = \frac{\beta_A}{\alpha_A + \beta_A}, \qquad \pi_{2,A} = \frac{\alpha_A}{\alpha_A + \beta_A}, \tag{3}$$

where α_A and β_A are transition probabilities from state 1 to state 2 and from state 2 to state 1, respectively.

The ACF of the mean process (2) of SPP reduces to

$$R_G(i) = \alpha_A \beta_A \left(\frac{G_{2,A} - G_{1,A}}{\alpha_A + \beta_A} \right)^2 (1 - \alpha_A - \beta_A)^i, \qquad i = 1, 2, \dots \tag{4}$$

where α_A and β_A are transition probabilities of the Markov modulating process from state 1 to state 2 and from state 2 to state 1, respectively. Note that $\lambda_A = (1 - \alpha_A - \beta_A)$ is the non-unit eigenvalue of the modulating Markov chain of SPP $\{W_A(n), n = 0, 1, \dots\}$. The values of λ_A and λ_G, α_A and α_G, β_A and β_G are the same since the modulating Markov chains of the mean process $\{W_G(n), n = 0, 1, \dots\}$ and SPP $\{W_A(n), n = 0, 1, \dots\}$ are the same.

The ACF of the SPP $\{W_A(n), n = 0, 1, \dots\}$ is expressed as follows [8]

$$R_A(i) = R_G(i) + E[W_A]\delta_n, \qquad \delta_n = \begin{cases} 1 & i = 0, \\ 0 & i = 1, 2, \dots \end{cases}, \tag{5}$$

where $E[W_A] = E[W_G] = \pi_{1,A} G_{1,A} + \pi_{2,A} G_{2,A}$ is the mean of SPP.

In order to completely parameterize the mean process of SPP, we must provide four parameters $(G_{1,A}, G_{2,A}, \alpha_A, \beta_A)$. If we choose $G_{1,A}$ as a free variable with constraint $G_{1,A} < E[W_A]$ to satisfy $0 < \lambda_A \leq 1$, we can determine $G_{2,A}$, α_A, and β_A from the next set of equations [12, 14, 8]

$$\begin{cases} G_{2,A} = \frac{D[X]}{E[X]-G_{1,A}} + G_{1,A} \\ \alpha_A = \frac{(1-K_X(1))(E[X]-G_{1,A})}{G_{2,A}-G_{1,A}} \\ \beta_A = \frac{(1-K_X(1))(G_{2,A}-E[X])}{G_{2,A}-G_{1,A}} \end{cases} . \quad (6)$$

where $\{X(n), n = 0, 1, \dots, N\}$ are observations of the covariance stationary arrival process, $D[X]$ is the variance of $\{X(n), n = 0, 1, \dots, N\}$, $E[X]$ is the mean of $\{X(n), n = 0, 1, \dots\}$, $K_X(1)$ is the lag-1 value of the normalized ACF (NACF). Parameters $(E[X], D[X], K_X(1))$ are estimated from empirical data as

$$\begin{cases} E[X] = \frac{\sum_{i=0}^{N} X(i)}{N+1} \\ D[X] = \frac{\sum_{i=0}^{N}(X(i)-E[X])^2}{N} \\ K_X(1) = \frac{\frac{1}{N-1}\sum_{i=0}^{N-1}(X(i)-E[X])(X(i+1)-E[X])}{D[X]} \end{cases} . \quad (7)$$

The reason to use SPP as a model of the frame arrival process is that its parameters are easily controllable [8]. From (6) one may note that there is a degree of freedom in choosing the mean arrival rate in state 1. Indeed, $G_{1,A}$ can take on any value from $(0, E[X])$ and still match the ACF and the mean of the arrival process. As a result, marginal distributions of these processes are different for different choices of $G_{1,A}$.

3 Error model

3.1 Stationarity of bit error observations

A bit error trace is essentially a sequence of successive events of correct and incorrect bit receptions at the physical layer. As usual, we assume that '1' represents an incorrectly received bit and '0' represents a correctly received bit. To use theory of stochastic processes, we consider a bit error trace as a realization of the stochastic bit error process $\{W_B(l), l = 0, 1, \dots\}$, $W_B(l) \in \{0, 1\}$, where l is the transmission time of a symbol on a wireless channel. We use $E[W_B]$ to denote the probability of bit error as seen by time-averages. The modeling process is denoted by $\{W_E(l), n = 0, 1, \dots\}$ with underlying Markov modulating process $\{S_E(l), n = 0, 1, \dots\}$.

Although no statistical studies have been carried out, most previous works implicitly assumed that the bit error trace is an observation of covariance stationary process. The concept of stationarity is an advantageous property of ergodic stochastic processes. Practically, if some stochastic observations are found to be non-stationary, their modeling is not usually feasible. A process is said to be strict stationary if its all M-dimensional distributions are the same. The class of strict stationary processes is a special class of covariance (weakly) stationary processes for which mean of all sections is the same and ACF depends on the time shift only.

While covariance stationarity for a whole trace may not always hold in practice, it was recently shown that covariance stationary segments of bit error traces can be isolated, analyzed and treated separately [15]. In this paper, following [15], we assume that a given bit error trace is either covariance stationary or can be segmented into a number of covariance stationary segments.

3.2 Related work

The first work that mathematically described the bit error process observed on a wireless channel is due to Gilbert [16]. His model has only two states of the modulating Markov chain, one of which is error-free while another one is associated with a non-zero bit error probability. Elliott [17] extended Gilbert's model allowing both states to have non-zero bit error probabilities. It is claimed that his model captures first-order statistics of the bit error process more precisely compared to Gilbert's one. The next extension came from Fritchman [18], who allowed a Markov chain to have more than one error-free state. In [19] authors extended previous works modeling bit error observations using a Markov chain with M, $M > 2$ states. Their model may have a non-zero bit error probability associated with each state. Authors also developed a parameters fitting algorithm to capture first-order statistics of the bit error observations. Recently, it was shown that these models may provide unsatisfactory results [20]. This is due to inherent nature of fitting algorithms used in [16, 17, 18, 19]. They capture first-order statistics neglecting memory properties of bit error observations.

There are also a number of models developed for layers, higher than physical, including data-link and IP layers. However, such models are limited to a given FEC capabilities of the data-link layer and cannot be effectively used to study performance response of wireless channels for different FEC codes. Indeed, since those models are limited to a given FEC code, for each particular FEC code a separate set of measurements is required to parameterize the wireless channel model.

Bit error models may provide the required versatility of the modeling environment. However, these models cannot be directly used in performance evaluation studies and must be properly extended to the layer of interest. For such an extension to be accurate, we must take into account specific peculiarities of underlying layers including statistical properties of involved stochastic processes, error concealment techniques at the data-link layer, segmentation procedures between different layers, etc. An adequate model of the wireless channel at the layer of interest is a complex cross-layer function of parameters of underlying layers and bit error stochastic process. These models along with traffic models have to be further applied to evaluate and optimize performance parameters experienced by applications running over wireless channels.

To our knowledge there have been no unified theoretical studies exploring the effect of bit error propagation to the data-link layer. In this paper we fill this gap investigating how the performance experienced by applications at the data-link layer is affected by first- and second-order properties of bit error statistics and different error correction capabilities of the data-link layer.

3.3 Bit error model

When MMBP is allowed to have two states only, and at most a single arrival is allowed in a slot, it reduces to switched Bernoulli process (SBP). Since this process has only two states of the modulating Markov chain, its ACF (2) is reduced to (4). NACF is then $K_G(i) = \lambda_E^i$, $i = 1, 2, \ldots$. It is clear that the NACF of the mean process of SBP exhibits geometrical decay that may produce fair approximation of empirical NACFs exhibiting nearly geometrical decay for small lags.

In our previous work [10, 21] we have shown that there is SBP model exactly matching mean and lag-1 autocorrelation of covariance stationary bit error observations. This model is given by

$$\begin{cases} \alpha_E = (1 - K_B(1))E[W_B] \\ \beta_E = (1 - K_B(1))(1 - E[W_B]) \end{cases}, \qquad \begin{cases} f_{1,E}(1) = 0 \\ f_{2,E}(1) = 1 \end{cases}, \tag{8}$$

where $f_{1,E}(1)$ and $f_{2,E}(1)$ are probabilities of error in states 1 and 2, respectively, α_E and β_E are transition probabilities from state 1 to state 2 and from state 2 to state 1, respectively, $K_B(1)$ is the lag-1 autocorrelation of bit error observations, $E[W_B]$ is the mean of bit error observations (bit error rate).

3.4 Frame error model

Assume that the length of frames is constant and equals to m bits. The sequence of consecutively transmitted bits, denoted by gray rectangles, is shown in Fig. 1, where $(n-1)$, n, $(n+1)$ denote time intervals whose length equals to the time to transmit a single frame; k, i, j, denote the state of the Markov chain $\{S_E(l), l = 0, 1, \dots\}$ in the beginning of these intervals.

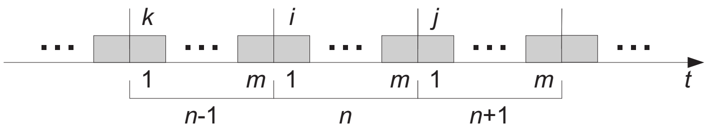


Figure 1: Sequence of consecutively transmitted bits at the wireless channel.

Consider the stochastic process $\{W_N(n), n = 0, 1, \dots\}$, $W_N(n) \in \{0, 1, \dots, m\}$, describing the number of incorrectly received bits in consecutive bit patterns of length m. This process is doubly stochastic, modulated by the underlying Markov chain $\{S_N(n), n = 0, 1, \dots\}$ and can be completely parameterized via parameters of the bit error process $\{W_E(l), l = 0, 1, \dots\}$ as shown below.

To parameterize $\{W_N(n), n = 0, 1, \dots\}$ we have to determine m-step transition probabilities between of the modulating Markov chain $\{S_E(l), l = 0, 1, \dots\}$ with exactly k, $k = 0, 1, \dots, m$, incorrectly received bits. Denote the probability of transition from the state i to state j for the Markov chain $\{S_N(n), n = 0, 1, \dots\}$ with exactly k, $k = 0, 1, \dots, m$ incorrectly received bits in a bit pattern of length m by $d_{N,ij}(k) = Pr\{W_N(n) = k, S_N(n) = j | S_N(n-1) = i\}$. Let the set of matrices $D_N(k)$, $k = 0, 1, \dots, m$ contain these transition probabilities. Matrices $D_N(k)$, $k = 0, 1, \dots, m$, can be found using $D_E(k)$, $k = 0, 1$, as follows

$$
\begin{aligned}
&D_N(0) = D_E^m(0),\\
&D_N(1) = \sum_{k=m-1}^{0} D_E^{m-k-1}(0) D_E(1) D_E^k(0),\\
&D_N(2) = \sum_{k=0}^{m-2} \left(D_E^k(0) D_E(1) \sum_{i=m-k-2}^{0} D_E^{m-i-k-2}(0) D_E(1) D_E^i(0) \right),\\
&\dots\\
&D_N(m-1) = \sum_{k=m-1}^{0} D_E^{m-k-1}(1) D_E(0) D_E^k(1),\\
&D_N(m) = D_E^m(1),
\end{aligned}
\tag{9}
$$

where $D_N(i)$, $i = 3, 4, \dots, m-2$ can be obtained by induction from $D_N(1)$ or $D_N(m-1)$. The easiest way is to induce $D_N(i)$, $i = 2, 3, \dots, \lfloor m/2 \rfloor$, from $D_N(1)$ and $D_N(i)$, $i = m-2, m-3, \dots, \lceil m/2 \rceil$ from $D_N(m-1)$. Note that computation according to (9) is still a challenging task. This becomes impossible when m is sufficiently large. Instead, one may use the recursive method as outlined below.

Let us extend the definition of $D_N(k)$, $k = 0,1,\dots$ as follows. We denote the probability of transition from the state i to state j for the Markov chain $\{S_N(n), n = 0,1,\dots\}$ with exactly k, $k = 0,1,\dots,m$ incorrectly received bits in a bit pattern of length m by $d_{N,ij}(k,m)$. Let the set of matrices $D_N(k,m)$, $k = 0,1,\dots,m$ contain these transition probabilities. Since at most two errors may occur in two consecutive slots, we have the following expression for $D_N(i,2)$, $i = 0,1,2$

$$D_N(i,2) = \sum_{k=0}^{i} D_E(k) D_E(i-k), \qquad i = 0,1,2, \tag{10}$$

where $D_E(2)$ is the matrix of zeros. Recursively, we get

$$\begin{aligned}
D_N(i,3) &= \sum_{k=0}^{i} D_N(k,2) D_E(i-k), & i &= 0,1,\dots,3,\\
D_N(i,4) &= \sum_{k=0}^{i} D_N(k,3) D_E(i-k), & i &= 0,1,\dots,4,\\
&\dots\\
D_N(i,m-1) &= \sum_{k=0}^{i} D_N(k,m-2) D_E(i-k), & i &= 0,1,\dots,m-1,\\
D_N(i,m) &= \sum_{k=0}^{i} D_N(k,m-1) D_E(i-k), & i &= 0,1,\dots,m,
\end{aligned} \tag{11}$$

where $D_E(k)$, $k \geq 2$ and $D_N(i,m)$, $i \geq m+1$ are all zero matrices. The latter equation in (11) gives $D_N(k)$, $k = 0,1,\dots$ for a given m.

Consider now the frame error process $\{W_F(n), n = 0,1,\dots\}$, $W_F(n) \in \{0,1\}$, where '0' indicates the correct reception of a frame, '1' denotes the incorrect frame reception. Process $\{W_F(n), n = 0,1,\dots\}$ is modulated by the underlying Markov chain $\{S_F(n), n = 0, 1, \dots\}$. Note that the state space of $\{S_F(n), n = 0, 1, \dots\}$ and $\{S_N(n), n = 0, 1, \dots\}$ is the same.

Let us denote the transition probability from state i to state j for the Markov chain $\{S_F(n), n = 0, 1, \dots\}$ with exactly k, $k = 0, 1$, incorrectly received frames by $d_{F,ij}(k)$, $k = 0, 1$. These probabilities are then combined in matrices $D_F(0)$ and $D_F(1)$. Process $\{W_N(n), n = 0, 1, \dots\}$, $W_N(n) \in \{0, 1, \dots, m\}$, describing the number of bit errors in consecutive frames is related to the frame error process $\{W_F(n), n = 0, 1, \dots\}$, $W_F(n) \in \{0, 1\}$, as follows

$$D_F(0) = \sum_{k=0}^{F_T - 1} D_N(k), \qquad D_F(1) = \sum_{k=F_T}^{m} D_N(k), \tag{12}$$

where F_T is the so-called frame error threshold determining how a frame is received. Expressions (12) are interpreted as follows: if the number of incorrectly received bits in a frame is greater or equal to a computed value of the frame error threshold ($k \geq F_T$), frame is incorrectly received and $W_F(n) = 1$. Otherwise ($k < F_T$), it is correctly received and $W_F(n) = 0$.

Assume that FEC is not used at the data-link layer. It means that every time a frame contains at least one bit error, it is received incorrectly. Thus, the transition probability matrices (12) of the frame error process take the following form

$$D_F(0) = D_N(0), \qquad D_F(1) = \sum_{k=1}^{m} D_N(k). \tag{13}$$

One may note that in (9) or (11) it is not required to compute $D_N(k)$ for all

$k = 1, 2, \ldots, m$. Alternatively, $D_F(1)$ can be found using the following relation

$$D_F(1) = \left(\sum_{k=0}^{m} D_N(k) \right) - D_N(0) = D_E^m - D_E^m(0). \tag{14}$$

Assume now that the number of bit errors that can be corrected by a FEC code in a frame of length m is l. Then, the frame error threshold is $F_T = (l+1)$ and the frame is incorrectly received whenever $k \geq F_T$. Otherwise, it is correctly received. Thus, the transition probability matrices (12) take the following form

$$D_F(0) = \sum_{k=0}^{F_T-1} D_N(k), \qquad D_F(1) = \sum_{k=F_T}^{m} D_N(k). \tag{15}$$

Again, in (15) we only need expressions for $D_N(k)$, $k = 0, 1, \ldots, F_T - 1$. Using the same reasoning as for (14), the expression for $D_F(1)$ is simplified as below

$$D_F(1) = D_E^m - \left(\sum_{k=0}^{F_T-1} D_N(k) \right). \tag{16}$$

One should note that the slot duration of $\{W_N(n), l = 0, 1, \ldots\}$ and $\{W_F(n), n = 0, 1, \ldots\}$ are the same, $\Delta t'$, and related to the slot duration of the bit error process $\{W_B(l), l = 0, 1, \ldots\}$ as $\Delta t' = m\Delta t$.

An illustration of the proposed cross-layer mapping for $F_T = 3$ is shown in Fig. 2, where time diagrams of $\{W_E(l), l = 0, 1, \ldots\}$, $\{W_N(n), n = 0, 1, \ldots\}$ and $\{W_F(n), n = 0, 1, \ldots\}$ are shown, where gray rectangles denote incorrect reception of PDUs at appropriate layers. Error threshold F_T must set as explained previously and then used to compute transition probability matrices of the frame error process at the data-link layer. The resulting frame error process is classified as D-MAP. Indeed, due to the cross-layer extension of the bit error model to the data-link layer, the transition probability matrices accompanied by incorrect and correct reception of a frame may have different probabilities in their rows.

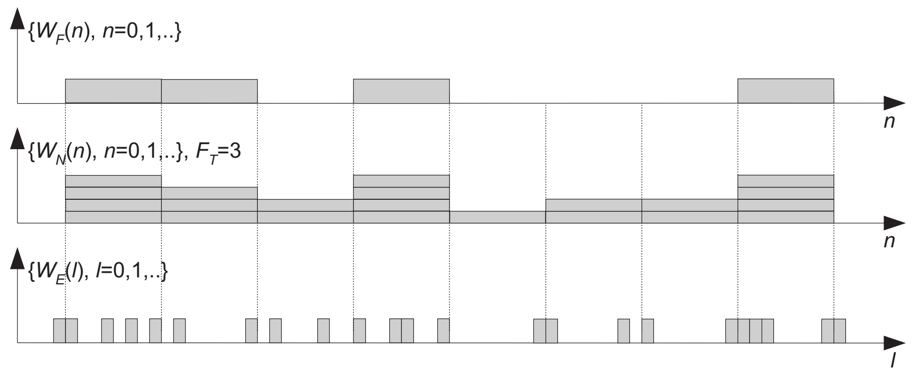
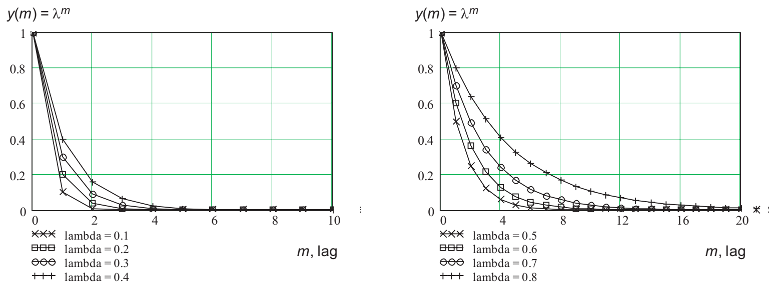


Figure 2: Illustration of the proposed cross-layer extension.

We note that the proposed cross-layer extension limits memory of the bit error process at lag m, where m is the length of the frame in bits. However, in practise, m is sufficiently large allowing to accurately capture memory of the initial bit error process. If ACF of the bit error process decays according to a single geometrical term, the threshold, $m_{\min}$, at which the frame error model becomes valid, can be computed using Fig. 3, where the function $y(m) = \lambda^m$ is plotted.

4 Performance model at the data-link layer

Up to date a number of performance evaluation models of the frame transmission process over wireless channels have been proposed. A comprehensive review can be found in [22]. In [22, 1] a performance evaluation model of the frame transmis-



(a) $\lambda = 0.1, 0.2, 0.3, 0.4$ (b) $\lambda = 0.5, 0.6, 0.7, 0.8$

Figure 3: Function $y(m) = \lambda^m$.

sion process over wireless channels was proposed. Our model combines accuracy of [23] and versatility of [6]. The latter property is due to queuing-theoretic origin of the model. Indeed, the model is benefit from a large set of specific solutions, algorithms, extensions and modifications developed to date in the broad scope of queuing and teletraffic theories. Accuracy stems from the accurate representation of wireless channel characteristics at the data-link layer. Indeed, consecutive bit errors are allowed to be autocorrelated or independent when necessary. The frame arrival process can be as arbitrary as D-BMAP. As a result, the proposed model allows to capture distributional and autocorrelational properties of the frame service and arrival processes. The solution of the problem involves the imbedded Markov chain approach resulting in two-dimensional Markov chain describing the queuing system at equilibrium.

4.1 Service process of the wireless channel

The straightforward way to represent the frame transmission process over a dedicated CBR wireless channel is to use $G_A/G_S/1/K$ queuing system, where G_A is the frame arrival process, G_S is the service process of the wireless channel, K

is the capacity of the system. Here, the service process is defined as times required to successfully transmit frames over a wireless channel. Characteristics of this process are heavily affected by the frame error process and error concealment schemes of the data-link layer.

It is known that both interarrival times of frames and transmission times of frames till successful reception can be autocorrelated. This property makes analysis of $G_A/G_S/1/K$ queuing system quite complex task even when arrival and service processes can be accurately modeled by Markovian processes. Indeed, theoretical background of queuing systems with autocorrelated arrival and service processes is not well-studied. Among few others, one should mention BMAP/SM/1 queuing system and some modifications considered in [24, 25, 26]. Analysis of such systems is more computationally intensive compared to queuing systems with renewal service processes. It usually involves imbedded Markov chains of large orders (larger than two). From this point of view, $G_A/G_S/1/K$ performance model does not provide significant improvements over other approaches.

4.1.1 Basic model for hybrid SW-ARQ/FEC and SR-ARQ/FEC

Consider the class of preemptive-repeat priority systems with two Markovian arrival processes. We allow both processes to have arbitrary autocorrelation structures of homogenous Markovian type. Assume that the first arrival process represents the frame arrival process from a traffic source. To provide adequate representation of unreliable transmission medium, we assume that the second arrival process is one-to-one mapping of the frame error process. That is, every time an error occurs, an arrival happens from this arrival process. In what follows, we refer to this process as the ’error arrival process’. An illustration of the mapping is shown in Fig. 4, where time evolution of the data-link layer wireless channel model and corresponding error arrival process is shown, black rectangles denote incorrect frame receptions, arrows indicate corresponding arrivals. Note that according to this mapping procedure, probabilistic properties of the stochastic model remain unchanged while interpretation of events changes. Making this process to be high priority one, and allowing its arrivals to interrupt ongoing service of low priority arrivals (those, from the frame arrival process), we assure that any arrival from this process immediately seizes the server for service, while the ongoing service is interrupted. A frame whose service is interrupted remains in the system (if allowed) and enters the server again after service completion of a high priority arrival. The service provided till the point of interruption is completely lost. It is interpreted as an incorrect reception of the frame from the traffic source and the priority discipline is referred to as preemptive-repeat.

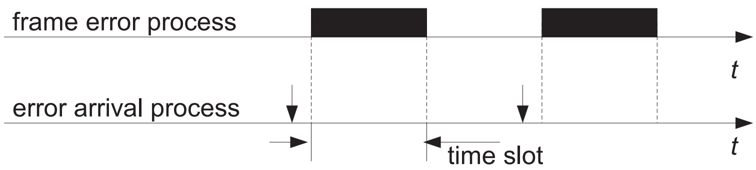


Figure 4: An illustration of error/arrival mapping.

To emulate behavior of SW-ARQ protocol, we assume an infinite number of retransmission attempts. We also assume that the feedback channel is completely reliable (perfect). Indeed, feedback acknowledgements are usually small in size and well protected by FEC code. Finally, we assume that the feedback is instantaneous. All these assumptions were tested and used in many studies and found to be appropriate for wireless channels [5, 27, 4, 28]. Since the wireless channel model is extended to the data-link layer, FEC capabilities are implicitly taken into account. Note that the described model is also suitable to represent 'ideal' selective-repeat ARQ (SR-ARQ) scheme as in [29, 30].

Analysis of queuing systems with priority discipline is still a challenging task. Among others, preemptive-repeat is probably most complicated priority discipline. However, a number of straightforward assumptions can be further introduced to make the queuing model less complicated. In what follows, we limit our model to the discrete-time environment and require arrivals from both arrival processes to have a service time of one slot in duration. Since frames at the data-link layer are usually of equal length, this assumption is not restrictive. According to such a system, arrivals occur just before the end of slots. Since there can be at most one arrival from the arrival process representing the frame error process of the wireless channel, these arrivals do not wait for service, enter the service in the beginning of nearest slots, and, if observed in the system, are being served. To provide adequate representation of erroneous nature of the wireless channel we also have to ensure that all arrivals from the error arrival process are accommodated by the system. Following these assumptions, it is no longer needed to require preemptive-repeat priority discipline. Since all arrivals occur simultaneously in batches, it is sufficient for such a system to operate according to non-preemptive priority discipline that is usually much easier to analyze.

4.1.2 Contention-free constant bit rate access

When a CBR channel is exclusively assigned to a mobile station during the whole duration of a session, to estimate performance parameters of the frame transmission process we can directly apply non-preemptive G_A+G_F/D/1/K queuing system, where G_A is the frame arrival process, G_F is the error arrival process. We assume that there is no data-link layer concurrent traffic competing for resources and the only performance degradation stems from unreliable nature of the wireless channel.

The sample path of the model is shown in Fig. 5, where transmission of frame and error arrivals are marked by grey and black rectangles, respectively, numbers are used to identify frames. One may note that the 'transmission of error frames' mimics the incorrect reception of data frames. To quantitatively study performance of the frame transmission process, we have to determine performance measures of the frame arrival process in G_A+G_F/D/1/K queuing system.

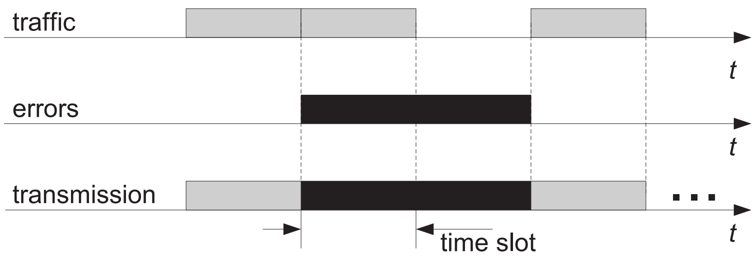


Figure 5: Sample path of the model for contention-free CBR access.

5 Performance evaluation

Consider the queuing system representing the frame service process of the wireless channel. The frame error model is a special case of D-MAP. We also assumed that the frame arrival process is represented by the special case of D-BMAP. In this section, we proceed with performance analysis of D-BMAP$_A$+D-MAP$_F$/D/1/K queuing system. Performance parameters of interest are probability functions of the number of lost frames and the delay of a frame. In what follows, the number of states of the modulating Markov chain of D-BMAP$_A$ and D-MAP$_F$ is allowed to be arbitrary finite, M_A and M_F, respectively.

5.1 Description of the system

Consider D-BMAP/D/1/K queuing system, where the arrival process, denoted by $\{W(n), n = 0, 1, \dots\}$, is the superposition of $\{W_F(n), n = 0, 1, \dots\}$ and $\{W_A(n), n = 0, 1, \dots\}$. Indeed, since both arrival processes are independent of each other one can define their superposition, that is again D-BMAP [11]. The counting variable n refers to the frame transmission time at the wireless channel. Steady-state analysis of D-BMAP/D/1/K queuing system has been carried out in many studies. Here, we take the method of imbedded Markov chain.

Time diagram of D-BMAP/D/1/K queuing system is shown in Fig. 6. According to such a system frames arrive in batches, batches of frames arrive just before the end of slots. Arrivals are not allowed to seize the server immediately and the service of any arrival starts at the beginning of a slot. Arrivals depart from the system at the slot boundaries, just after batch arrivals (if any). The state of the system is observed just after the departure (if any) and these points are imbedded Markov points. This system is known as 'late arrival model with delayed access' [31, 32]. The sojourn (service) time is counted as the number of slots spent by a frame in the system. The system can accommodate at most K frames. We assume partial batch acceptance strategy. According to this strategy, if a batch of R frames arrives when k frames are in the system and $R > (K - k)$, only $(K - k)$ frames are accommodated and $(R - K + k)$ frames are discarded.

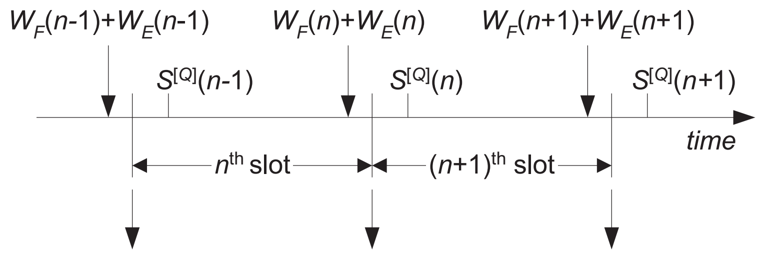


Figure 6: Time diagram of D-BMAP/D/1/K discrete-time queuing system.

5.2 Steady-state distribution of D-BMAP/D/1/K

The following equation relating the number of frames in the system between successive imbedded Markov points is the fundamental part of the imbedded Markov chain analysis

$$S_Q(n+1) = \max\left(0, S_Q(n) - 1\right) + \min\left(W(n+1), K - S_Q(n)\right), \qquad (17)$$

where $S_Q(n)$ denotes the number of frames (from both processes) in the system, $W(n)$ denotes the number of arrivals in the slot n.

Observing (17) and Fig. 6, it can be deduced that the arrival from the frame error process is not accepted by the system in the slot $(n+1)$ if and only if the number of customers in the system in the slot $(n-1)$ is zero, there is an arrival of K frames in the time slot n, and one frame arrives from the frame error process in the slot $(n+1)$. Contrarily, if there is at least one frame in the system in the slot $(n-1)$, one frame departs at the boundary between slots n and $(n+1)$, and there is always at least one position in the system for the next arrival. Thus, the frame from the frame error process (if any) is not lost in the slot $(n+1)$. To assure that the frame from the frame error process is always accepted by the system we do not allow the overall number of arrivals from both processes to be more than $(K-1)$. This implies that the maximum number of arrivals from the frame arrival process is $(K-2)$, that is usually sufficient for real applications.

Complete description of the queuing system requires two-dimensional Markov chain $\{S_Q(n), S(n), n = 0, 1, \dots\}$ imbedded at the moments of frame departures from the system, where $S(n) = S_A(n) \otimes S_F(n)$ is the state of superposition of the frame arrival and frame error processes, and $S_Q(n) \in \{0, 1, \dots, K-1\}$ is the number of frames in the system just after frame departures. Introducing matrices $D(\geq k)$, $k = 0, 1, .., K-1$, containing transition probabilities with at least $k = 0, 1, .., K-1$ arrivals, respectively, one can define the transition probability matrix, T, of the Markov chain $\{S_Q(n), S(n), n = 0, 1, \dots\}$ as usual (see [33] among many others). Let $\vec{x} = (x_{0,1}, .., x_{K-1,M})$ be the row array containing steady-state probabilities of $\{S_Q(n), S(n), n = 0, 1, ..\}$. Solving matrix equations $\vec{x}T = \vec{x}$, $\vec{x}\vec{e} = 1$, one can compute steady-state probabilities $x_{kj} = \lim_{n\rightarrow\infty} Pr\{S_Q(n) = k, S(n) = j\}$. There are a number of algorithms to compute these probabilities [34, 35, 36].

5.3 Loss performance

Let us derive the expression for probability function of the number of lost PDUs arriving from the PDU arrival process. Since we guaranteed that the PDU error process does not suffer losses, from the loss performance point of view D-BMAP$_A$+D-MAP$_F$/D/1/K and D-BMAP/D/1/K queuing systems, where D-BMAP is the superposition of D-MAP$_F$ and D-BMAP$_A$, are equivalent. Consider the loss behavior of D-BMAP/D/1/K queuing system between two arbitrary imbedded Markov points at equilibrium. Since at most $(K-2)$ PDUs may arrive from the PDU arrival process, there can be at most $(K-2)$ lost PDUs in a slot. Let the RV L, $L \in \{0, 1, \ldots, K-2\}$, denote the number of lost PDUs in a slot and let $f_L(l) = Pr\{L(n) = l | W_A(n) \geq 1\}$, $l = 0, 1, \ldots, K-2$, be its probability function.

Consider the event when a single PDU from the PDU arrival process is lost in an arbitrary slot n at equilibrium. Since at most $(K-2)$ arrivals are allowed from the PDU arrival process, this process does not suffer losses when there are less than two PDUs in the system in the slot $(n-1)$. Hence, exactly one PDU from the PDU arrival process is lost when the following conditions are simultaneously met:

- there are k, $k = 2, 3, \ldots$ PDUs in the system in the slot $(n-1)$;
- there are exactly $(K-k+1)$ arrivals to the system in the slot n.

Taking into account these conditions over all possible transitions of the underlying Markov chain of the superposed arrival process with exactly $(K-k+1)$ arrivals from both processes we get the following

$$f_L(1) = \frac{\sum_{k=3}^{K-1} \sum_{i=1}^{M} \sum_{j=1}^{M} x_{ki} d_{ij}(0, K-k+1)}{Pr\{W_A(n) \geq 1\}} + \\ + \frac{\sum_{k=2}^{K-1} \sum_{i=1}^{M} \sum_{j=1}^{M} x_{ki} d_{ij}(1, K-k)}{Pr\{W_A(n) \geq 1\}}, \quad (18)$$

where $d_{ij}(l, k)$, $l = 0, 1$, $k = 0, 1, \ldots, K-2$ are transition probabilities from the state i to state j of the superposed arrival process accompanied by l arrivals from the PDU error process and k arrivals from the PDU arrival process, x_{ki}, $k = 0, 1, \ldots, K-1$, $i = 1, 2, \ldots, M_F M_A$ are the steady state probabilities that there are k PDUs in the system and the state of the arrival process is i, $Pr\{W_A(n) \geq 1\}$ is the probability that at least one PDU arrives from the PDU arrival process. Since the system never reaches states (K, i), $i = 1, 2, \ldots, M$, the first sums in (18) extend to $(K - 1)$ only. Next sums in (18) cover all possible states of the underlying Markov chain of the superposed arrival process in previous and next slots.

Similarly as in (18) we get the following expression for l, $l = 1, 2, \ldots, K - 2$, lost PDUs in an arbitrary slot n at equilibrium

$$f_L(l) = \frac{\sum_{k=3}^{K-1} \sum_{i=1}^{M} \sum_{j=1}^{M} x_{ki} d_{ij}(0, K - k + l)}{Pr\{W_A(n) \geq 1\}} + \\ + \frac{\sum_{k=2}^{K-1} \sum_{i=1}^{M} \sum_{j=1}^{M} x_{ki} d_{ij}(1, K - k + l - 1)}{Pr\{W_A(n) \geq 1\}}. \quad (19)$$

Alternatively, in matrix notation we may write

$$f_L(l) = \frac{\sum_{k=3}^{K-1} \vec{x}_k D(0, K - k + l)\vec{e}}{Pr\{W_A(n) \geq 1\}} + \\ + \frac{\sum_{k=2}^{K-1} \vec{x}_k D(1, K - k + l - 1)\vec{e}}{Pr\{W_A(n) \geq 1\}}, \quad (20)$$

where $D(l, k)$, $l = 0, 1$, $k = 0, 1, \ldots, K - 2$ are transition probability matrices of the superposed arrival process with exactly l arrivals from the PDU error process and j arrivals from the PDU arrival process, $\vec{x}_k = (x_{k1}, x_{k2}, \ldots, x_{k(M_F M_A)})$ is the vector containing steady-state probabilities that there are k PDUs in the system and the state of the modulating Markov chain of the superposed arrival process is $i = 1, 2, \ldots, M_F M_A$, $\vec{e}$ is the vector of ones of appropriate size.

Expression for $D(l,k)$ can be obtained from the superposed arrival process as

$$D(l,k) = D_F(l) \otimes D_A(k), \qquad l = 0,1, \qquad k = 0,1,\dots. \tag{21}$$

The probability function of the number of lost PDUs obtained in (19) and (20) is conditioned on the event of at least one arrival from the PDU arrival process. We find this condition of (19) and (20) as follows

$$Pr\{W_A \geq 1\} = \vec{\pi}_A \left(\sum_{i=1}^{K-2} D_A(i) \right) \vec{e}_A, \tag{22}$$

where $\vec{\pi}_A$ is the steady-state vector of the modulating Markov chain of the PDU arrival process, $\vec{e}_A$ is the vector of ones of appropriate size.

Finally, we have

$$\begin{aligned} f_L(l) = & \frac{\sum_{k=3}^{K-1} \vec{x}_k D(0, K-k+l)\vec{e}}{\vec{\pi}_A \left(\sum_{i=1}^{K-2} D_A(i) \right) \vec{e}_A} + \\ & + \frac{\sum_{k=2}^{K-1} \vec{x}_k D(1, K-k+l-1)\vec{e}}{\vec{\pi}_A \left(\sum_{i=1}^{K-2} D_A(i) \right) \vec{e}_A}, \qquad l = 1,2,\dots,K-2, \\ f_L(0) = & 1 - \sum_{i=1}^{K-2} f_L(i). \end{aligned} \tag{23}$$

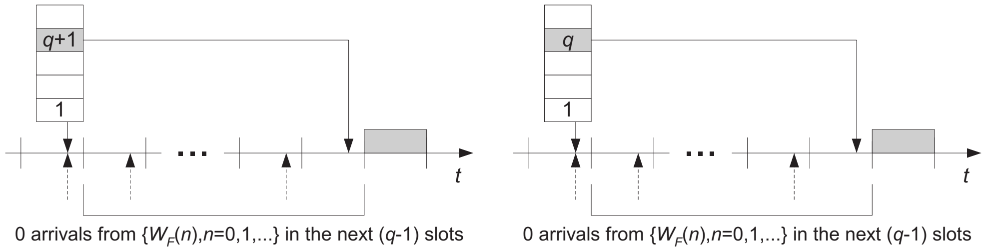
Moments of loss distribution can be directly obtained from (23).

5.4 Delay performance

Let us derive the expression for probability function of the delay that a PDU from the PDU arrival process arriving and accommodated by the system, experiences while transmitting over a wireless channel. To do so consider D-BMAP$_A$+D-

MAP_F/D/1/K queuing system at equilibrium and observe an arbitrary slot n. We tag an arbitrary arrival from the PDU arrival process that arrives in the slot n and accommodated by the system. Let the random variable Q, $Q \in \{1, 2, \dots\}$ denote the full delay in the system (sojourn time) experienced by this arrival and let $f_Q(q) = Pr\{Q(n) = q|W_A(n) > L(n)\}$, $q = 1, 2, \dots$ be its probability function, where $W_A(n)$ is the number of arriving PDUs in the slot n, $L(n)$ is the number of lost PDUs in the slot n. Note that the delay suffered by the tagged arrival is the sum of the service time and the time it spends in the buffer. Since arrivals from the PDU error process are always served first, the maximum delay, that the tagged arrival may experience in the system is virtually unlimited.

Consider the tagged PDU arriving from the PDU arrival process and accommodated by the system in an arbitrary slot n at equilibrium. If there is at least one PDU in the system in the slot $(n - 1)$ and the tagged arrival is accommodated by the system at the position $(q + 1)$ (including the PDU being served) in the slot n and during the following $(q-1)$ slots there are no arrivals from the PDU error process, the tagged arrival is taken for service at the boundary between slots $(q - 1)$ and q, served uninterruptedly during the slot q and successfully departs from the system at the boundary between slots q and $(q + 1)$. In this case the waiting time is q slots, where during the first $(q - 1)$ slots the arrival is waiting in the buffer and during the slot q the arrival is served. This situation is illustrated in Fig. 7(a). The waiting time is also q slots, if there are no PDUs in the system in the slot $(n - 1)$, the tagged arrival is accommodated by the system at the position q in the slot n and during the following $(q - 1)$ slots there are no arrivals from the PDU error process as shown in Fig. 7(b). One may note that in both cases the delay experienced by the tagged arrival is conditioned on arrivals from the PDU error process in the next $(q-1)$ slots.



(a) $S_Q(n-1) \neq 0$ (b) $S_Q(n-1) = 0$

Figure 7: Illustration of the conditional delay experienced by the tagged arrival.

Let $f_Q(q, 0)$, $q = 1, 2, \ldots, K-1$, be the vectors containing probabilities that the tagged PDU arriving in the slot n is at the position q just after the slot boundary between slots n and $(n+1)$ and the state of the superposed arrival process is j given that at least one PDU arriving from PDU arrival process is not lost. These vectors are defined as follows

$$f_Q(q, 0) = (Pr\{I(n) = q, S(n) = j | W_A(n) > L(n)\}; j = 1, 2, \ldots, M), \tag{24}$$

where $I(n)$ is the place of the tagged PDU just after the slot boundary. If there are no arrivals from the PDU error process during the waiting time of the tagged arrival, sum of elements in $f_Q(q, 0)$ can be interpreted as the conditional probability of waiting time.

Consider the case when $f_Q(1, 0)$. This happens to the tagged arrival arriving in the slot n when the number of PDUs in the system in the slot $(n-1)$ is $S_Q(n-1) = 0$, at least one PDU arrives from the PDU arrival process in the slot n, the tagged arrival is not lost and accommodated by the system at the 1st position in the buffer.

This also occurs when there is only one PDU in the system in the slot $(n-1)$, at least one PDU arrives from the PDU arrival process in the slot n, the tagged arrival is not lost and accommodated at the second position in the system. In the latter case one PDU departs from the system at the boundary between slots $(n-1)$ and n and the tagged arrival still suffers one slot delay. These two conditions result in the following expression

$$f_Q(1,0) = \frac{\sum_{i=1}^{K-2} \vec{x}_0 D(0,i)\psi_{K,i} + \sum_{i=1}^{K-2} \vec{x}_1 D(0,i)\psi_{K-1,i}}{Pr\{W_A(n) > L(n)\}}, \qquad (25)$$

where $\psi_{v,i}$ is the probability that the tagged arrival is accommodated at the place i in the system when there are v waiting positions available for arrivals from the PDU arrival process, $Pr\{W_A(n) > L(n)\} = Pr\{W_A(n) \geq 1\} - Pr\{L(n) = W_A(n) \geq 1\}$ is the probability that at least one arrival is not lost in the slot n, $Pr\{L(n) = W_A(n) \geq 1\}$ is the probability that all arrivals from the PDU arrival process in the slot n are lost.

Since all arrivals from the PDU arrival process have the same priority, the probability $\psi_{v,i}$ is independent of the actual position of the tagged arrival among other arrivals from the PDU arrival process. It has uniform distribution over all accommodated arrivals from the PDU arrival process in a batch, and given by

$$\psi_{v,i} = \frac{1}{\min(v,i)}, \qquad v = 0,1,\ldots,K, \qquad i = 1,2,\ldots,K-1. \qquad (26)$$

Consider the event when all PDUs arriving from the PDU arrival process in the slot n are lost. Since K PDUs cannot be observed in the system at the imbedded Markov points, this event occurs when there are $(K-1)$ PDUs in the system in the slot $(n-1)$, one arrival occurs from the error arrival process in the slot n and at least one PDU arrives from the PDU arrival process. We have

$$Pr\{L(n) = W_A(n) \geq 1\} = \sum_{i=1}^{K-2} \vec{x}_{K-1} D(1,i)\vec{e}. \tag{27}$$

Using the same reasoning as in (25) we get

$$\begin{aligned} f_Q(2,0) = & \frac{\sum_{i=2}^{K-2} \vec{x}_0 D(0,i)\psi_{K,i} + \sum_{i=2}^{K-2} \vec{x}_1 D(0,i)\psi_{K-1,i}}{\vec{\pi}_A \left(\sum_{i=1}^{K-2} D_A(i)\right)\vec{e}_A - \sum_{i=1}^{K-2} \vec{x}_{K-1} D(1,i)\vec{e}} + \\ & + \frac{\sum_{i=1}^{K-2} \vec{x}_0 D(1,i)\psi_{K-1,i} + \sum_{i=1}^{K-2} \vec{x}_1 D(1,i)\psi_{K-2,i}}{\vec{\pi}_A \left(\sum_{i=1}^{K-2} D_A(i)\right)\vec{e}_A - \sum_{i=1}^{K-2} \vec{x}_{K-1} D(1,i)\vec{e}} + \\ & + \frac{\sum_{i=1}^{K-2} \vec{x}_2 D(0,i)\psi_{K-2,i}}{\vec{\pi}_A \left(\sum_{i=1}^{K-2} D_A(i)\right)\vec{e}_A - \sum_{i=1}^{K-2} \vec{x}_{K-1} D(1,i)\vec{e}}. \end{aligned} \tag{28}$$

Finally, we get the following for $f_Q(q,0)$, $q = 1, 2, \ldots, K-1$

$$\begin{aligned} f_Q(q,0) = & \frac{\sum_{i=q}^{K-2} \vec{x}_0 D(0,i)\psi_{K,i} + \sum_{i=q-1}^{K-2} \vec{x}_0 D(1,i)\psi_{K-1,i}}{\vec{\pi}_A \left(\sum_{i=1}^{K-2} D_A(i)\right)\vec{e}_A - \sum_{i=1}^{K-2} \vec{x}_{K-1} D(1,i)\vec{e}} + \\ & + \frac{\sum_{k=1}^{q} \sum_{i=q-k+1}^{K-2} \vec{x}_k D(0,i)\psi_{K-k,i}}{\vec{\pi}_A \left(\sum_{i=1}^{K-2} D_A(i)\right)\vec{e}_A - \sum_{i=1}^{K-2} \vec{x}_{K-1} D(1,i)\vec{e}} + \\ & + \frac{\sum_{k=1}^{q-1} \sum_{i=q-k}^{K-2} \vec{x}_k D(1,i)\psi_{K-k-1,i}}{\vec{\pi}_A \left(\sum_{i=1}^{K-2} D_A(i)\right)\vec{e}_A - \sum_{i=1}^{K-2} \vec{x}_{K-1} D(1,i)\vec{e}}. \end{aligned} \tag{29}$$

Let us recall that the waiting time of the tagged arrival in D-BMAP$_F$+D-MAP$_A$/D/1/K depends on whether arrivals from the PDU error process happen during its waiting time. Consider the event when the tagged arrival suffers q, $q = 1, 2, \ldots$ slots delay. This event occurs when and only when the following conditions are simultaneously met:

- tagged arrival is accommodated at the position:
  - $i+1$, $i \leq q$, in the slot n when $S_Q(n-1) \neq 0$;
  - i, $i \leq q$, in the slot n when $S_Q(n-1) = 0$.
- there are $(q-i)$ arrivals from $\{W_F(n), n = 0, 1, \dots\}$ in next $(q-2)$ slots;
- there is no arrival from $\{W_F(n), n = 0, 1, \dots\}$ in the slot $(n+q-1)$.

The first term has been found in (29), the last is given by

$$D(0, \cdot) = \sum_{i=0}^{K-2} D(0, i). \tag{30}$$

Using (30) and introducing $D(l, \cdot) = \sum_{j=0}^{K-2} D(l, j)$, $l = 0, 1$ we can find transition probability matrices $T(i, m)$, $i = 0, 1, \dots, m$, $i \leq m$, with exactly i arrivals from $\{W_F(n), n = 0, 1, \dots\}$ in m, $m = 1, 2, \dots$ successive slots, starting from the slot $(n+1)$. These matrices are given by

$$\begin{aligned}
&T(0, m) = D^m(0, \cdot), \\
&T(1, m) = \sum_{k=m-1}^{0} D^{m-k-1}(0, \cdot) D(1, \cdot) D^k(0, \cdot), \\
&\dots \\
&T(m-1, m) = \sum_{k=m-1}^{0} D^{m-k-1}(1, \cdot) D(0, \cdot) D^k(1, \cdot), \\
&T(m, m) = D^m(1, \cdot),
\end{aligned} \tag{31}$$

where $T(i, m)$, $i = 3, 4, \dots, m$ can be obtained by induction from $T(2, m)$ or $T(m-2, m)$. The easiest way is to deduct $T(i, m)$, $i = 3, 4, \dots, \lfloor m/2 \rfloor$, from

$T(2,m)$ and $T(i,m)$, $i =, m, m-1, \ldots, \lceil m/2 \rceil$ from $T(m-2,m)$. Note that computation according to (31) is a challenging task involving significant complexity. Alternatively, the following recursion can be used

$$
\begin{aligned}
T(i,2) &= \sum_{k=0}^{i} D(k,\cdot)D(i-k,\cdot), && i = 0,1,\ldots 2, \\
T(i,3) &= \sum_{k=0}^{i} T(k,2)D(i-k,\cdot), && i = 0,1,\ldots,3, \\
&\ldots \\
T(i,m-1) &= \sum_{k=0}^{i} T(k,m-2)D(i-k,\cdot), && i = 0,1,\ldots,m-1, \\
T(i,m) &= \sum_{k=0}^{i} T(k,m-1)D(i-k,\cdot), && i = 0,1,\ldots,m, \qquad (32)
\end{aligned}
$$

where $D(l,\cdot)$, $l \geq 2$ and $T(k,m)$, $k \geq m+1$ are all zero matrices.

Using the convolution of (29) and (32) we get the following expression for probability function of the delay suffered by the tagged arrival from the PDU arrival process

$$
\begin{aligned}
f_Q(1) &= f_Q(1,0)\vec{e} \\
f_Q(2) &= f_Q(2,0)D(0,\cdot)\vec{e} \\
f_Q(q) &= \sum_{i=2}^{q} f(i,0)T(q-i,q-2)D(0,\cdot)\vec{e}, \quad q = 3,4,\ldots,K-1, \\
f_Q(q) &= \sum_{i=2}^{K-1} f(i,0)T(q-i,q-2)D(0,\cdot)\vec{e}, \quad q = K,K+1,\ldots, \qquad (33)
\end{aligned}
$$

where $\vec{e}$ is the unit vector of appropriate size.

Moments of the PDU delay can be readily obtained from (33).

6 Numerical results

In this section numerical examples for the data-link layer are provided. We firstly study how the performance response of the wireless channel in terms of probability functions of the number of lost frames, the delay of a frame and their first moments varies for different bit error statistics, frame arrival statistics and FEC schemes. Then, we consider two particular FEC codes and discuss how mean values of the number of successfully transmitted data bits in a slot and the delay of a frame varies for a wide range of bit error statistics.

6.1 Bit error models

To explore the performance response of the wireless channel we use a number of SBP wireless channel models with different means and lag-1 autocorrelations. We constructed 90 models of the bit error process as follows. For each mean out of $E[W_E] \in \{0.1, 0.02, \dots, 0.09\}$ we generated models with the following lag-1 autocorrelations $K_E(1) \in \{0.0, 0.1, \dots, 0.9\}$. Parameters of the bit error models, α_E and β_E, as a function of $E[W_E]$ and $K_E(1)$ are shown in Fig. 8.

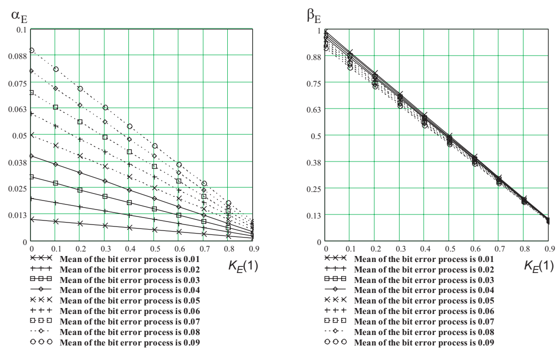


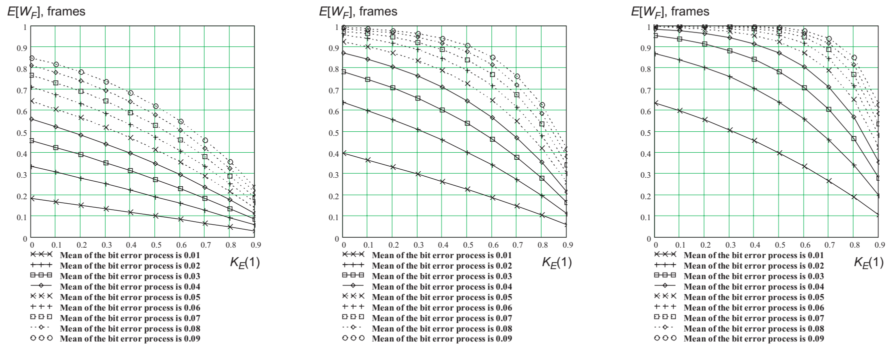
Figure 8: Parameters α_E and β_E as a function of $E[W_E]$ and $K_E(1)$.

6.2 Frame error models

To use the bit error models in performance evaluation studies of the frame transmission process they were extended to the data-link layer taking into account FEC correction capabilities. This was done according to Section 3.

The frame error probability for all bit error models and frame sizes $m = 20, 50, 100$ without FEC ($F_T = 1$) are shown in Fig. 9. Observing these figures one may note that qualitative characteristics (trends) are the same for all m. The difference is quantitative, caused by parameters of the bit error models and values of m. As expected, with increase in the bit error rate the frame error probability increases for all values of lag-1 autocorrelation. Noticeably, that big values of lag-1 autocorrelation results in significantly better performance, especially for big values of the bit error rate. Thus, for $m > 20$, no FEC environment and a given bit error rate weak memory of the wireless channel should lead to worse performance of applications due to higher frame error rate. This justifies the thesis that the increase in the linear dependence between successive bit error observations does not lead to worse quality of the wireless channel. As illustrated in Fig. 9 with increase in the frame size the frame error probability increases leading to worse channel conditions as compared to smaller size of frames. As a result, even for relatively small bit error rate the channel may become completely unusable for large values of m when no FEC is used at the data-link layer. Therefore, the performance of the wireless channel can be significantly improved using the small frame size and FEC codes.

Consider now how FEC procedures affect the mean value of the frame error process at the data-link layer. The frame error probability for $(20, 1)$, $(20, 2)$,

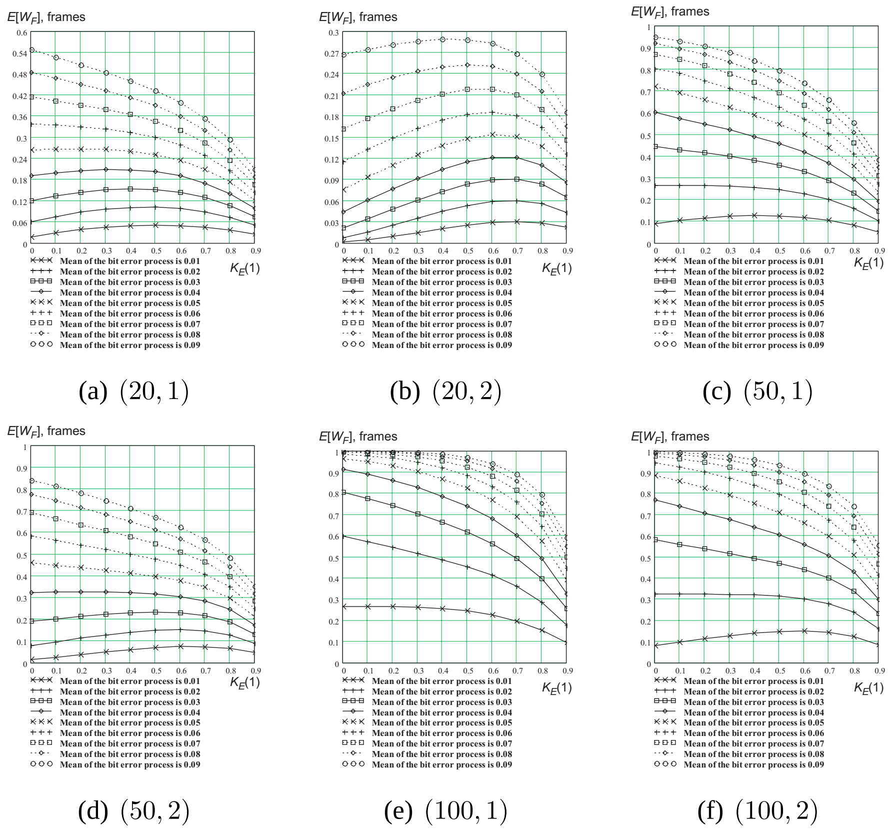


(a) $m = 20$ (b) $m = 50$ (c) $m = 100$

Figure 9: Frame error probability as a function of m.

(50,1), $(50, 2)$, (100,1) and $(100, 2)$ FEC configurations and for all defined bit error models is shown in Fig. 10. The influence of parameters of the bit error process is specific for a certain (m, l). As expected, for all values of $E[W_E]$ and $K_E(1)$ the frame error rate becomes smaller with introduction of FEC. When the FEC code is weak (e.g. $(50, 1)$, $(100, 1)$, $(100, 2)$) and the bit error rate is high (for example, higher than 0.03 for $(50, 1)$) the increase in the lag-1 autocorrelation always results in smaller values of $E[W_F]$. When the FEC code is stronger, small and high values of lag-1 autocorrelation lead to better channel quality in terms of the frame error rate as compared to moderate values of $K_E(1)$ (e.g. $0.4 \sim 0.6$ for $(20, 1)$). As expected, the increase in $E[W_E]$ results in increase of $E[W_F]$. However, this dependency is not straightforward.

Summarizing, we state that the frame error probability strictly depends on the interplay between the bit error probability, linear dependence between successive bit error observations and a given FEC code. The optimal frame size for a given FEC code and wireless channel characteristics can be computed using the proposed approach and taking into account the number of data bits for each particular FEC scheme. In what follows, we use the following (m, l) pairs: $(20, 0)$,



(a) $(20, 1)$ (b) $(20, 2)$ (c) $(50, 1)$

(d) $(50, 2)$ (e) $(100, 1)$ (f) $(100, 2)$

Figure 10: Frame error probability as a function of (m, l).

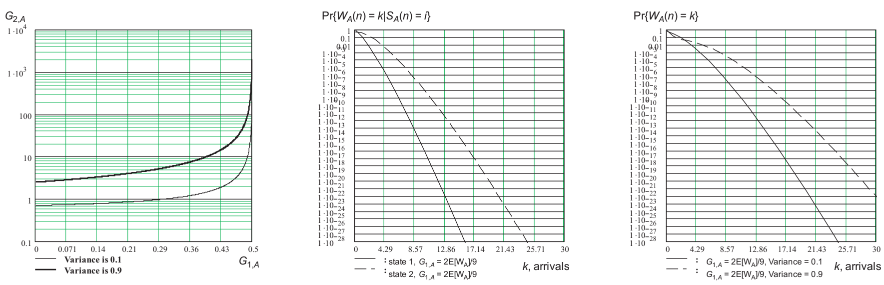
$(20, 1)$, $(20, 2)$. Recall that the number of data bits in a single frame decreases when the number of bits that can be successfully corrected increases.

6.3 Frame arrival models

To explore the performance response of the wireless channel we use a number of SPP frame arrival models with different marginal distributions and lag-1 autocorrelations. In what follows, the SPP frame arrivals process is denoted by $\{W_A(n), n = 0, 1, \dots\}$ and its mean process is denoted by $\{W_G(n), n = 0, 1, \dots\}$. We constructed 90 models of the frame arrival process as follows.

Firstly, for a constant mean of the process set to $E[W_A] = E[W_G] = 0.5$ and mean in the state 1 set to $G_{1,A} = 2E[W_A]/9$, the variance of the mean arrival process of SPP is different and given by $D[W_G] \in \{0.1, 0.2, \dots, 0.9\}$. Note that for constant $E[W_A]$ and $G_{1,A}$, change in the value of variance affects the mean number of arrivals in the state 2. Note that the variance of SPP and its mean process are different. However, the difference is $E[W_A]$ and it is kept constant for all models. Therefore, the increase in the value of variance of the mean process increases the variance of SPP. Recall that the primary goal is to explore the qualitative effect of arrival process characteristics on its service performance.

Fig. 11(a) demonstrates how the mean in the state 2 of SPP$_A$ varies with changes in the value of variance for $E[W_A] = E[W_G] = 0.5$, $G_{1,A} = 2E[W_A]/9$. As a result, even for constant $E[W_A]$ and $G_{1,A}$, distributions of SPP$_A$ are different due to the effect of variance. As an example, probability functions of the number of arrivals for both states of the arrival process are shown in Fig. 11(b), where $E[W_A] = 0.5$, $G_{1,A} = 2E[W_A]/9$, $D[W_G] = 0.5$. Probability functions of the number of arrivals for two SPP$_A$ with $D[W_G] = 0.1$ and $D[W_G] = 0.9$ are shown in Fig. 11(c).



(a) $G_{2,A} = f(G_{1,A})$ (b) PFs for states 1 and 2 (c) PFs of SPP$_A$

Figure 11: Various dependencies in characteristics of SPP$_A$.

To study the effect of the lag-1 autocorrelation of the frame arrival process on performance response of the wireless channel, for each value of $D[W_G]$ we generated models with the following lag-1 autocorrelations $K_G(1) \in \{0.0, 0.1, \ldots, 0.9\}$. Parameters of these models, α_A and β_A, as a function of $D[W_G]$ and $K_G(1)$ for $E[W_A] = 0.5$ and $G_{1,A} = 2E[W_A]/9$ are shown in Fig. 12.

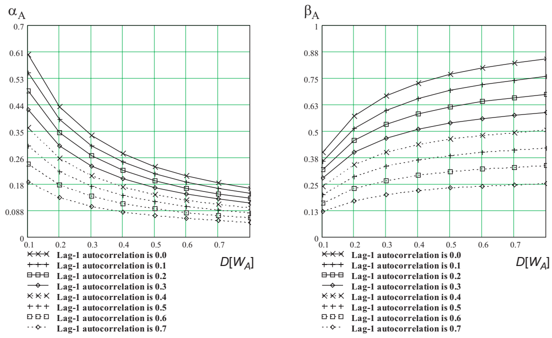


Figure 12: Parameters α_A and β_A as a function of $D[W_G]$.

6.4 Results

The resulting system is SPP$_A$+SBP$_F$/D/1/K that is a special case of D-BMAP$_A$+D-MAP$_F$/D/1/K. In what follows, we set capacity of the system to $K = 40$. Recall that we should not allow the frame arrival process to have more than $(K - 2)$ arrivals in a slot. Values of $Pr\{W_A(n) = 38|S_A(n) = 2\}$ for different values of $G_{1,A}$ corresponding to each value of $D[W_G] \in \{0.1, 0.2, \ldots, 0.9\}$ are shown in Fig. 13. One may note that the probability of 38 arrivals in a slot is very small (almost impossible) and can be neglected. Indeed, even for $G_{2,A}$ corresponding to $G_{1,A} = 2E[W_A]/9$ and $D[W_G] = 0.9$, $Pr\{W_A(n) = 38|S_A(n) = 2\} = 2.038E - 27$. Note that $Pr\{W_A(n) = 38|S_A(n) = 1\}$ is constant for all values of $D[W_G]$ and equal to $9.404E - 82$. Since at most one arrival is allowed from the frame error process, the requirement $Pr\{S_Q(n) = K, S(n) = j|S_Q(n-1) =$

$0, S(n-1) = i\} = 0$ is satisfied.

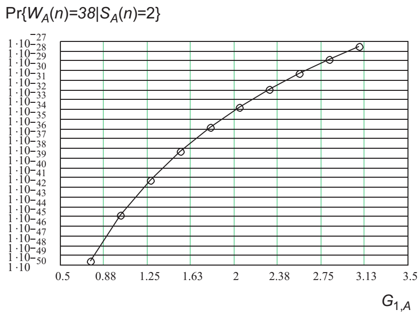


Figure 13: $Pr\{W_A = 38|S_A(n) = 2\}$ for different values of $D[W_G]$.

6.4.1 Probability functions

Let us firstly consider how performance parameters at the data-link layer are affected by first- and second-order statistical characteristics of the bit error process. The effect of the lag-1 autocorrelation of the bit error process is shown in Fig. 14, where probability functions of the number of lost frames and delay of a frame are depicted for different values of $K_E(1)$. The mean of the bit error process was set to 0.03, parameters of the frame arrival process were $E[W_A] = 0.5$, $D[W_G] = 0.5$, $K_G(1) = 0.0$, $G_{1,A} = 2E[W_A]/9$, and the FEC code was $(20, 2)$. One may see that the lag-1 autocorrelation of the bit error process does affect loss and delay performance experienced at the data-link layer. The increase in $K_E(1)$ results in increase of probabilities corresponding to large waiting times in the system. The probabilities of any non-zero number of lost frames are higher for large values of the lag-1 autocorrelation of the bit error process. Note that for small values of $K_E(1)$ associated probability functions of the delay of a frame have a knee when q reaches the capacity of the system (in our case $K = 40$).

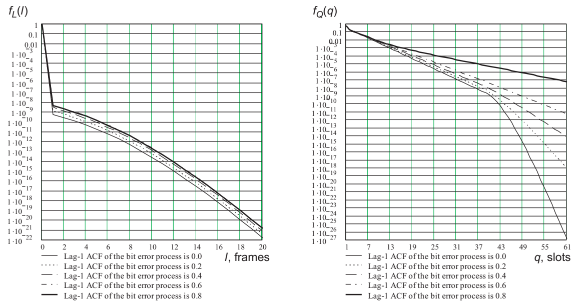


Figure 14: The effect of the lag-1 autocorrelation of the bit error process.

The effect of the mean of the bit error process is shown in Fig. 15 for $K_E(1) = 0.0$, $E[W_A] = 0.5$, $K_G(1) = 0.0$, $D[W_G] = 0.5$, $G_{1,A} = 2E[W_A]/9$ and FEC code $(20, 2)$. As expected, the increase in the bit error rate leads to significantly worse performance in terms of both delays and losses.

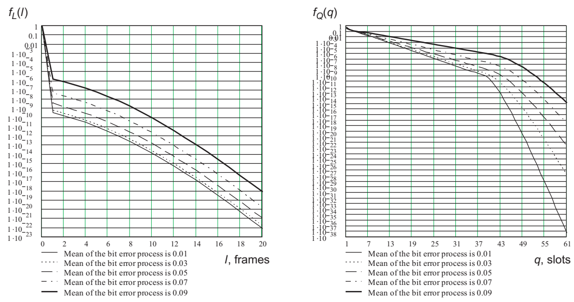


Figure 15: The effect of the mean of the bit error process.

Consider now how first- and second-order statistical characteristics of the frame arrival process affect performance parameters experienced at the data-link layer. Probability functions of the number of lost frames and the delay of a frame for different values of $K_G(1)$ and $K_E(1) = 0.0$, $E[W_E] = 0.03$, $E[W_A] = 0.5$, $D[W_G] = 0.5$, $G_{1,A} = 2E[W_A]/9$ are shown in Fig. 16. One can see that the lag-1 autocorrelation may drastically change the performance experienced at the data-link layer. Particularly, the increase in $K_G(1)$ results in substantial increase of probabilities corresponding to any non-zero number of lost frames. Therefore, the arrival process with higher memory performs worse than that with weaker linear dependence between successive arrivals. Such a profound effect of lag-1 autocorrelation coefficient is due to relatively large arrival rate. It was found that smaller mean arrival rate results in less significant effect. The effect of the lag-1 autocorrelation of the frame arrival process on delay performance at the data-link layer is more complicated. For small values of the waiting time (up to $5 - 7$) higher values of $K_G(1)$ lead to lower probabilities of the waiting time as compared to smaller values of $K_G(1)$. The effect of the lag-1 autocorrelation is opposite for larger waiting times.

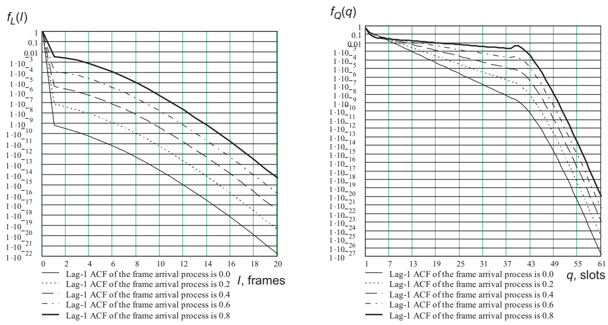


Figure 16: The effect of the lag-1 autocorrelation of the frame arrival process.

Finally, the influence of the distribution of the number of arriving frames is illustrated in Fig. 17 and Fig. 18. In Fig. 17 the variance of the mean frame arrival process of SPP$_A$ was changed while other parameters were set to $K_G(1) = 0.0$, $E[W_A] = 0.5$, $G_{1,A} = 2E[W_A]/9$, $E[W_E] = 0.03$, $K_E(1) = 0.0$. One can see that the increase in variance of the frame arrival process leads to significant increase of probabilities corresponding to any non-zero number of lost frames and significant increase of probabilities corresponding to large waiting times in the system. In Fig. 18 we vary the form of the distribution changing $G_{1,A}$ from

$2E[W_A]/9 = 0.111$ to $8E[W_A]/9 = 0.444$ and keep the variance of the mean frame arrival process of SPP_A constant and equal to 0.5. Note that the difference in probability functions of the number of lost frames is noticeable for these values of $G_{1,A}$.

Summarizing, we conclude that changes in the form of the distribution of the number of arriving frames does affect the performance at the data-link layers in terms of the probability functions of the number of lost frames and delay of a frame. This property should be taken into account in performance evaluation and control of wireless channel performance.

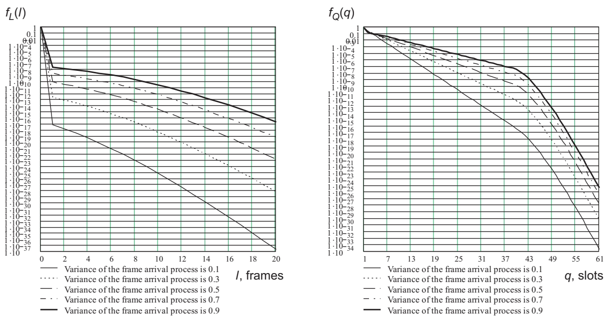


Figure 17: The effect of the variance of the frame arrival process.

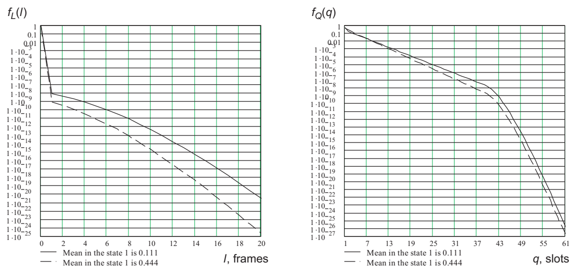


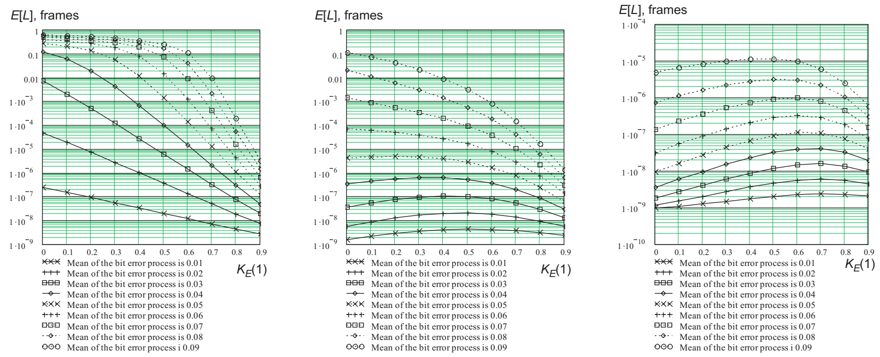
Figure 18: The effect of $G_{1,A}$.

6.4.2 Mean values

The mean number of lost frames and mean delay of a frame are convenient parameters that are often used to describe the quality perceived at the data-link layer. Consider how these parameters change in response to changes in first- and second-order bit error and frame arrival statistics and correction capability of FEC codes.

Firstly, consider how the mean loss and delay performance of the frame transmission process is affected by changes in first- and second-order statistics of the bit error process for different FEC capabilities. Fig. 19 shows how the mean loss response varies with changes in mean and lag-1 autocorrelation of the bit error process for the following FEC capabilities: $(20,0)$, $(20,1)$ and $(20,2)$. Parameters of the frame arrival process were set to $E[W_A] = 0.5$, $K_G(1) = 0.0$, $D[W_G] = 0.5$ and $G_{1,A} = 2E[W_A]/9$. When the FEC code is not used the increase in the value of lag-1 autocorrelation always leads to smaller mean number of lost frames. However, when the FEC code is used, small and high values of $K_E(1)$ result in better loss performance as compared to moderate values of $K_E(1)$ (e.g. $K_E(1) = 0.3 \sim 0.6$) for small values of bit error rate. Details of this effect strictly depend on the FEC capability of the data-link layer. As expected, the increase in the bit error rate always leads to worse performance in terms of the mean number of lost frames. The difference, however, becomes smaller for all FEC capabilities as the memory of the wireless channel increases.

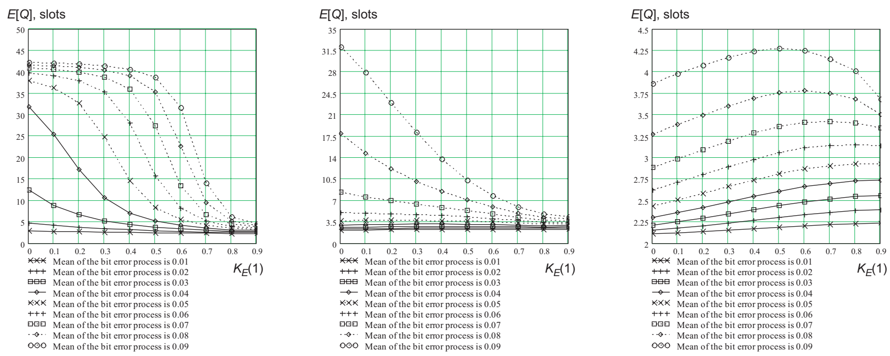
The mean delay response at the data-link layer for different bit error rates and lag-1 autocorrelation of the bit error process and FEC capabilities is shown in Fig. 20. For $(20,1)$ FEC code or when the FEC code is not used, the increase in the value of lag-1 autocorrelation leads to better mean delay performance at the data-



(a) $(20, 0)$ (b) $(20, 1)$ (c) $(20, 2)$

Figure 19: The mean loss response for different (m, l), $K_E(1)$, $E[W_E]$.

link layer. Depending on the FEC capability and a given bit error rate this trend can be faster or slower than linear. However, for stronger FEC capabilities (e.g. $(20, 2)$) the increase in $K_E(1)$ results in worse delay performance of the wireless channel. This effect depends on the bit error rate of the wireless channel. As expected, the increase in the bit error rate leads to worse mean delay response. However, the magnitude of this effect strictly depends on the value of lag-1 autocorrelation. For example, when the FEC code is not used, this effect is negligible for extremely big values of $K_E(1)$.



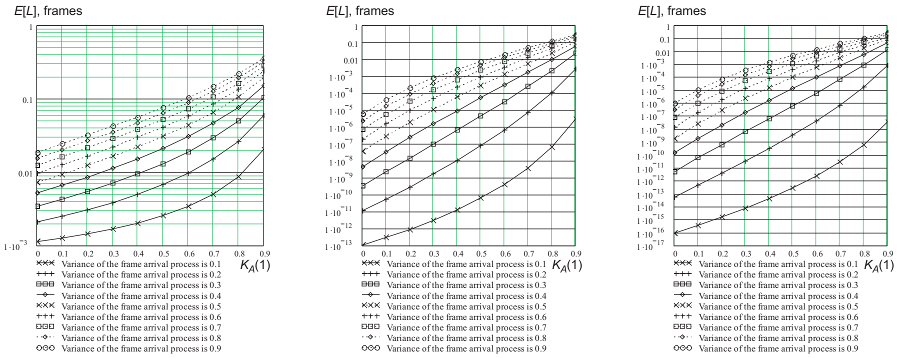
(a) $(20, 0)$ (b) $(20, 1)$ (c) $(20, 2)$

Figure 20: The mean delay response for different (m, l), $K_E(1)$, $E[W_E]$.

Summarizing the effect of the channel statistics, we conclude that the mean delay and loss performance at the data-link layer are indeed sensitive to both fist- and second-order statistics of the bit error process and correction capability of the FEC code. Particularly, not only the bit error rate but the memory properties of the wireless channel should also be taken into account when estimating loss and delay performance at the data-link layer with ARQ and FEC procedures.

Consider now whether the mean loss and delay response at the data-link layer are affected by first- and second-order order statistics of the frame arrival process. The mean number of lost frames as a function of the variance and lag-1 autocorrelation of the mean frame arrival process of SPP_A for different FEC capabilities is shown in Fig. 21. Other parameters of the frame arrival and bit error processes were set to $E[W_E] = 0.03$, $K_E(1) = 0.0$, $E[W_A] = 0.5$ and $G_{1,A} = 2E[W_A]/9$. For all FEC capabilities the increase in lag-1 autocorrelation leads to worse mean loss performance at the data-link layer. The increase in the variance of the mean frame arrival process of SPP_A also leads to worse loss performance even for the same mean number of arrivals. Depending on particular values of $D[W_G]$ and $K_G(1)$, both trends can be faster or slower than linear.

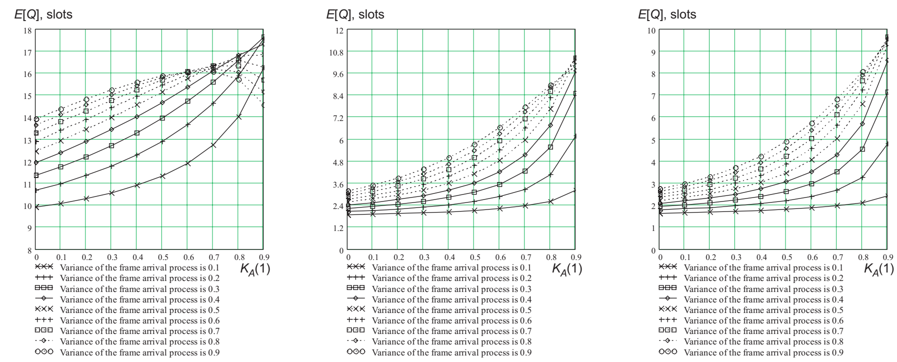
The mean delay response for the same parameters of arrival and error processes and FEC codes is shown in Fig. 22. For all FEC capabilities the increase in lag-1 autocorrelation almost always leads to worse mean delay performance. However, when the FEC code is not used large values of $K_G(1)$ may lead to better delay performance at the data-link layer for big values of $D[W_G]$. This is due to the fact that in these examples we do not directly control parameters of SPP. All we change are parameters of the mean process of SPP. In Fig. 22(a) we affect the value of $K_Y(1)$ by changing both $K_G(1)$ and $D[W_G]$. The resulting effect on



(a) $(20, 0)$ (b) $(20, 1)$ (c) $(20, 2)$

Figure 21: The mean loss response for different (m, l), $D[W_G]$, $K_G(1)$.

$K_Y(1)$ is not straightforward. The increase in the value of $K_G(1)$ results in increase of $K_Y(1)$. However, when $D[W_G]$ increases change in the value of $K_Y(1)$ depends on the interplay between $D[W_G]$ and $D[W_Y]$. Indeed, the value of $D[W_Y]$ also increases in response to the increase in the value of $D[W_G]$. The increase in variance almost always results in worse mean delay performance except for no FEC environment where large values of $K_G(1)$ and $D[W_G]$ lead to better mean delay performance.



(a) $(20, 0)$ (b) $(20, 1)$ (c) $(20, 2)$

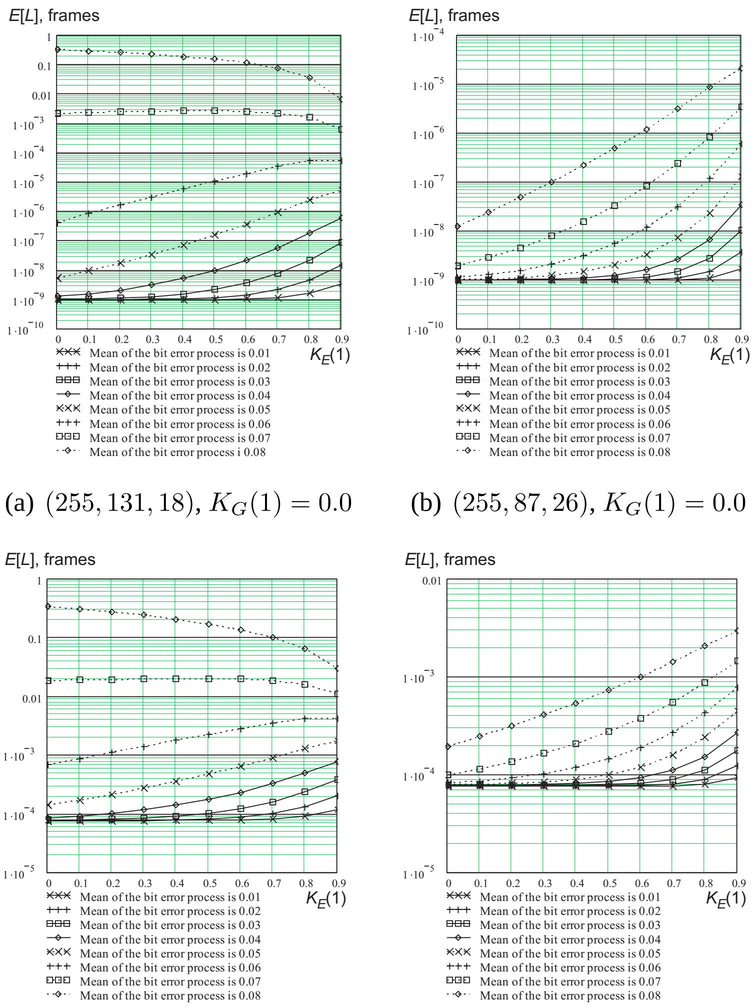
Figure 22: The mean delay response for different (m, l), $D[W_G]$, $K_G(1)$.

Summarizing, the mean loss and delay performance at the data-link layer are also sensitive to first- and second-order statistics of the frame arrival process. As a result, the interplay between statistical characteristics of the bit error and frame arrival processes is of paramount importance for optimal choice of the FEC code.

6.4.3 Case study

In this case study we use two particular Bose-Chaudhuri-Hochquenghem (BCH) FEC codes denoted by triplet (m, n, l), where m is the length of the frame in bits, n is the number of data bits in a frame and l is the number of incorrectly received bits that can be corrected. The codes are $(255, 131, 18)$ and $(255, 87, 26)$, whose rates are approximately $1/2$ and $1/3$, respectively. According to our model, the former coding scheme tries to deliver approximately 1.507 times more data bits in a single slot.

Fig. 23 demonstrates the mean loss performance for both FEC codes, different wireless channel conditions and different statistics of the arrival process. Other parameter of the frame arrival process were set to $E[W_A] = 0.5$, $D[W_G] = 0.5$, $G_{1,A} = 2E[W_A]/9$. If the performance metric of interest is the mean number of successfully delivered frames, one can see from Fig. 23 that $(255, 87, 26)$ FEC code outperforms $(255, 131, 18)$ FEC code for all conditions of the wireless channel. However, if the performance metric of interest is the mean number of successfully delivered bits, the result can be different. This stems from the fact that $(255, 131, 18)$ FEC code tries to transmit approximately 1.5 times more data bits in a single slot. As a result, a new comparison should be made. The mean number of successfully delivered bits in a slot can be computed as $(E[W_A(n)|W_A(n) \geq 1] - E[L(n)|W_A(n) \geq 1])n$, where n is the number of data bits in a single frame. For $K_G(1) = 0.0$, it can be computed that $(255, 87, 26)$ FEC code is only better when the bit error rate is 0.08 and the lag-1 autocorrelation is less than 0.4. The performance at the data-link layer in terms of both metrics is significantly worse when the lag-1 autocorrelation of the mean frame arrival process increases from 0.0 to 0.5. Note that the final decision on the choice of the FEC code should also take into account the mean delay performance of both FEC codes as explained below.



(a) $(255, 131, 18)$, $K_G(1) = 0.0$ (b) $(255, 87, 26)$, $K_G(1) = 0.0$

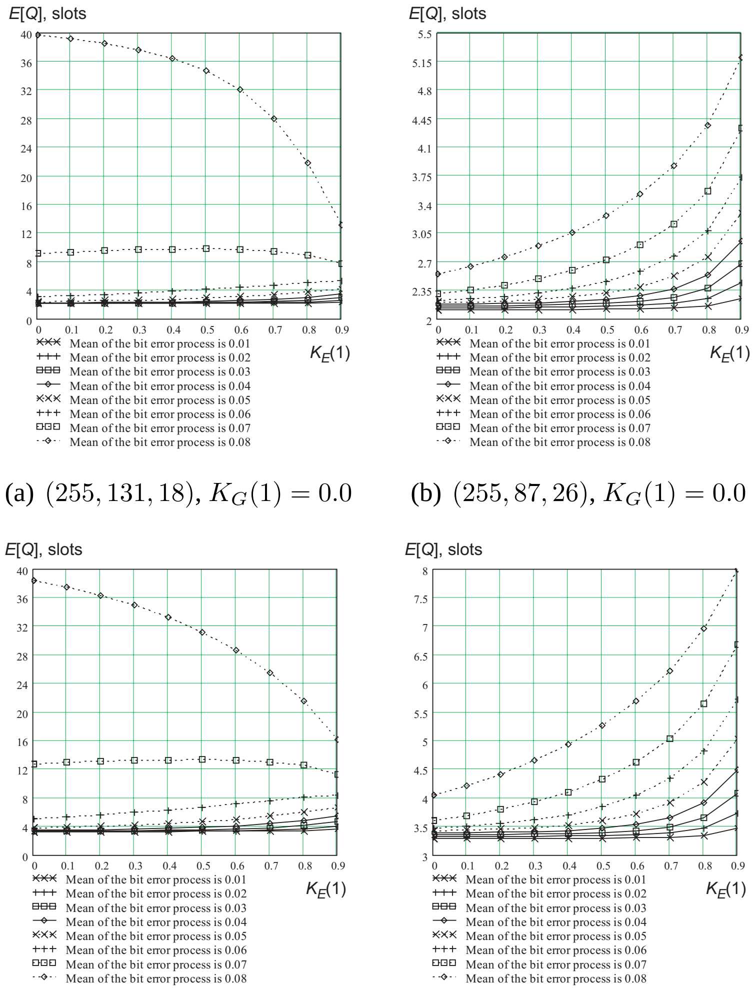
(c) $(255, 131, 18)$, $K_G(1) = 0.5$ (d) $(255, 87, 26)$, $K_G(1) = 0.5$

Figure 23: Case study: the mean loss response for different FEC codes.

Fig. 24 illustrates the delay experienced by a frame when $(255, 131, 18)$ and $(255, 87, 26)$ FEC codes are used at the data-link layer. Recall that in real environment ARQ protocols sometimes limit the number of retransmission attempts. For example, if the number of retransmission attempts is limited by 6, ARQ protocol with $(255, 131, 18)$ FEC code does not perform well on a wireless channel with the bit error rate higher than 0.07. For most considered wireless channel conditions, ARQ protocol with $(255, 87, 26)$ FEC code performs better than with $(255, 131, 18)$ FEC code. For example, for $E[W_E] = 0.08$, $K_E(1) = 0.4$, $K_G(1) = 0.0$ approximately 3 retransmission attempts are required on average. For the same wireless channel conditions and traffic parameters ARQ protocol with $(255, 131, 18)$ FEC code requires around 36 retransmission attempts which is not tolerable. Note that when the lag-1 autocorrelation of the mean frame arrival process of SPP$_A$ increases to $K_G(1) = 0.5$ and the wireless channel conditions are $E[W_E] = 0.08$ and $K_E(1) = 0.4$, even $(255, 87, 26)$ FEC code leads to poor performance as it now requires approximately 5 retransmission attempts to deliver a single frame correctly.

7 Conclusions

In this paper we developed an analytical cross-layer performance evaluation approach for wireless channel with autocorrelated arrival and loss processes. We use lag-1 autocorrelation coefficient and mean bit error rate as a description of wireless channel conditions. The bit error process was then extended to the data-link layer using the cross-layer mapping procedure that allows to explicitly take into account FEC and ARQ procedures. The frame arrival process was modeled by



(a) $(255, 131, 18)$, $K_G(1) = 0.0$ (b) $(255, 87, 26)$, $K_G(1) = 0.0$

(c) $(255, 131, 18)$, $K_G(1) = 0.5$ (d) $(255, 87, 26)$, $K_G(1) = 0.5$

Figure 24: Case study: the mean delay response for different FEC codes.

D-BMAP. Queuing-theoretic approach was then used to derive performance parameters of interest including probability functions of the number of lost frames and delay of a frame. The proposed methodology allows to take into account both first- and second-order channel and traffic statistics and still analytically derive performance parameters of the frame transmission process avoiding computationally intensive simulations. It also allows to easily change the strength of the FEC code to determine best possible performance for a given traffic and channel conditions without the need for extensive measurements of wireless channel characteristics for each particular FEC code.

Using the proposed approach we observed that the loss and delay response of wireless channels is highly sensitive to first- and second-order channel and arrival statistics. Particularly, both bit error rate and lag-1 autocorrelation may significantly change the performance perceived by applications at the data-link layer. The performance response is also sensitive to the form of the probability function of the number of arriving frames. The interplay between statistical characteristics of bit error and frame arrival processes is then of paramount importance for optimal choice of FEC code. Failure to model traffic and channel characteristics appropriately may lead to misleading expectations regarding the performance level provided by a wireless channel.

The proposed approach is a versatile tool combining characteristics of propagation environment, traffic parameters and local error correction capabilities of the wireless channel in a single mathematical bundle that provides performance of a certain application as an output. Although the approach does not seem to be appropriate for on-line implementation as an element of the performance control system, it is still much faster than simulation studies. For example, it can be used to implement the so-called bank of performance curves that can be used at both ends of the wireless interface to provide the best possible performance at any instant of time for current wireless channel conditions. Finally, the proposed approach can also be extended to the IP layer taking into account segmentation of a single IP packet into a number of frames (codewords) at the data-link layer.

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