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Abstract

The *Universal Mobile Telecommunication System* (UMTS) operates with *Wideband Code Division Multiple Access* (WCDMA) over the air interface. Most studies dealing with the capacity of CDMA networks consider the uplink (reverse link) and evaluate the coverage or capacity of a cell or network. This focus on the uplink originates in the fact that the IS-95 network was a single voice network such that the network performance was limited by the uplink. Furthermore, fast power control was then implemented on the uplink only. The introduction of 3G networks leaves the pure voice networks behind and instead provides a variety of different services which by majority produce asymmetric traffic with the bulk on the downlink (forward link). This traffic asymmetry shifts the capacity limit from the uplink to the downlink. The 3gpp standard prescribes the use of fast power control for the downlink, as well. In this paper we propose a model to calculate the first and second moment of the NodeB transmit power which allows us to approximate its distribution and to determine the probability that the system becomes instable when a certain transmit power is exceeded.

1 Introduction

The imminent introduction of the *Universal Mobile Telecommunication System* (UMTS) in Europe demands a sophisticated network planning. This requires among others to answer the following questions: First, does the network cover the desired area. And second, does the network carry the offered traffic or, in other words, is the network capacity sufficient. The coverage area is on the uplink limited by the maximum transmit power of a mobile and on the downlink by the maximum NodeB power per dedicated channel which is significantly larger. Therefore, the uplink is considered to be the limiting factor for the coverage area, [1]. The uplink capacity is limited by the amount of multiple access interference at the NodeBs, the downlink by the available NodeB power. In the literature the amount of work on the downlink capacity lags behind the publications on the uplink capacity by far. The reasons for this trace back to the early cdmaOne systems and are two-fold: First, the cdmaOne systems supported only voice traffic which results in a symmetric traffic distribution between uplink and downlink. This shifts the capacity bottleneck to the uplink. Second, in the early CDMA systems fast-power control was defined for the uplink only. The introduction of 3G systems working with WCDMA leads or is expected to lead to applications with asymmetric traffic, in particular the browsing in the World Wide Web (WWW) or audio and video-streaming. These applications produce on the downlink a multiple of the uplink traffic and consequently shift the capacity limit towards the downlink. The evaluation of the downlink capacity mostly relies on Monte Carlo simulation, see e.g. [2, 3]. The authors of [4] analyze the downlink capacity of a CDMA system with mixed multi-rate sources in a multi-path fading channel and obtain the outage probability. By introducing a downlink power factor, which is the ratio of other-cell interference to same-cell interference averaged over the cell, they gain the outage probability and Erlang capacity in closed form. Their analysis, however, does not consider the traffic processes at the surrounding cells. Instead, they assume that all base station transmit with an equal total power.

The authors of [5] and [6] propose a Monte Carlo simulation and an analytic method to evaluate the capacity of a general UMTS network with arbitrary NodeB layout, a heterogeneous spatial traffic distribution, and multiple services. In [7] this approach is extended for the downlink using an iterative method to determine the distribution of the NodeBs' transmit powers. In this paper we propose a simpler method to approximate the transmit powers by computing the first and second moment through a matrix inversion.

The rest of the paper is organized as follows: Section 2 defines the power control equation for the downlink and formulates the considered problem exactly. Sections 3 and 4 are dedicated to the Monte Carlo simulation and the analytic method. Section 5 presents two example scenarios and Section 6 concludes the paper.

2 Problem Formulation

The limiting factor for the downlink capacity of a UMTS network is the transmit power required from the Node-B to maintain the E_b/N_0-requirements of all users. On the uplink the received powers of all users of the same service class at the same Node-B are equal assuming perfect power control. On the downlink, however, the transmit and also the received powers at the mobiles depend on the mobiles' locations. Further, the codes of the mobiles belonging to one Node-B are orthogonal to each other such that in an ideal case the mobiles do not interfere with each other. Due to multi-path propagation, however, the perfect orthogonality is lost and a part of the power is seen as interference. The orthogonality factor α defines the share of the power received by a mobile of the same Node-B that contributes to the interference. Hence, mobile k receives the signal from Node-B x with an E_b/N_0 of

$$\hat{\varepsilon}_{k,x} = \frac{W}{R_k} \cdot \frac{\hat{S}_{k,x} \cdot \hat{d}_{x,k}}{W\hat{N}_0 + \sum\limits_{y \neq x} \hat{S}_y \hat{d}_{y,k} + \alpha \left(\hat{S}_x - \hat{S}_{k,x} \right)}, \tag{1}$$

where $\hat{d}_{x,k}$ is the propagation gain. The transmit power $\hat{S}_x$ of Node-B x corresponds to the sum of the transmit powers $\hat{S}_{k,x}$ of all mobiles with a connection to Node-B x plus a constant part $\hat{S}_{x,C}$ for common channels.

The aim of our study is to derive a method that allows us to determine whether a UMTS network is able to carry the offered traffic on the downlink. A UMTS network consists of L Node-Bs with fixed positions. The offered traffic follows a spatial traffic distribution. The considered area $\mathcal{F}$ is subdivided into smaller squares f and each of these squares offers a Poisson distributed number of users with density a_f. Further, the UMTS network provides S different services and p_s is the probability that a users operates with service s. A service is defined by its bit rate R_s and its E_b/N_0-target ε_s^*. The combination of the UMTS network, the spatial traffic distribution, and the service mixture we call a scenario. In the following, we derive the probability that the transmit power $\hat{S}_x$ exceeds a given maximum $\hat{S}_{x,max}$ for Node-B x. Note that we denote a linear value by $\hat{a}$ while a is in decibels. Though the meaning is different on the uplink we refer to it as outage probability. We denote the thermal noise density by $N_0 = -174 dBm/Hz$ and the system bandwidth by $W = 3.84 Mcps$. We propose three different methods to compute this probability: a Monte Carlo simulation, a "direct" method and a more sophisticated method called "state".

3 Monte Carlo Simulation

The Monte Carlo simulation generates a series of system snapshots according to the scenario and determines the NodeBs' transmit powers. A snapshot consists of a set of mobiles with their service and position. The propagation gain $\hat{d}_{x,k}$ from NodeB x to a mobile k follows from the propagation model. The method to compute the power allocations according to Eq. (1) is similar to the methods in [5] for the uplink and in [7] for the downlink.

Assume that mobile k belongs to NodeB x, or short $k \in x$. Then the transmit power $\hat{S}_{x,k}$ follows from Eq. (1):

$$\hat{S}_{k,x} = \left(\omega_{k,0} W \hat{N}_0 + \sum_{y \neq x} \omega_{k,y} \hat{S}_y + \omega_{k,x} \hat{S}_x \right) \tag{2}$$

We define the service dependent load ω_k of a mobile k and it's service and position dependent load $\omega_{k,y}$ where y corresponds to the own NodeB x, another NodeB or the thermal noise:

$$\omega_k = \frac{\hat{\varepsilon}_k R_k}{W + \alpha \hat{\varepsilon}_k R_k} \text{ and } \omega_{k,y} = \begin{cases} \omega_k \frac{1}{\hat{d}_{x,k}} & \text{if } y = 0 \\ \omega_k \alpha & \text{if } y = x \\ \omega_k \frac{\hat{d}_{y,k}}{\hat{d}_{x,k}} & \text{if } y \neq x \end{cases} . \tag{3}$$

Further, if we sum over all mobiles belonging to NodeB x we speak of the load $\eta_{x,y}$ for NodeB x related to y and define this load as

$$\eta_{x,y} = \sum_{k \in x} \omega_{k,y} \text{ and } \eta_x = \sum_{k \in x} \omega_k . \tag{4}$$

The total transmit power $\hat{S}_x$ of NodeB x consists of a constant part $\hat{S}_{x,C}$ required for common channels and the variable part spent for the dedicated channels to the mobiles belonging to x:

$$\hat{S}_x = \hat{S}_{x,C} + \eta_{x,0} W \hat{N}_0 + \sum_y \hat{S}_y \eta_{x,y} \tag{5}$$

These equations for all NodeBs x are written as a matrix equation and solved for the vector $\bar{S} = (\hat{S}_1, ..., \hat{S}_L)^T$

$$\begin{aligned} \bar{S} &= \bar{S}_C + \bar{\eta}_0 W \hat{N}_0 + \tilde{\eta} \bar{S} \\ \Leftrightarrow \bar{S} &= \left(\tilde{E} - \tilde{\eta}\right)^{-1} \left(\bar{S}_C + \bar{\eta}_0 W \hat{N}_0\right) \end{aligned} \tag{6}$$

Note, that a variable $\bar{v}$ stands for a vector and a variable $\tilde{m}$ for a matrix. So $\bar{\eta}_0 = (\eta_{1,0}, ...\eta_{L,0})^T$, $\bar{S}_C = (\hat{S}_{1,C}, ..., \hat{S}_{L,C})^T$, $\tilde{\eta}$ is the $L \times L$-matrix with $\tilde{\eta}(x, y) = \eta_{x,y}$, and $\tilde{E}$ is the $L \times L$-identity matrix. A reasonable solution exists if the inverse of the matrix $(\tilde{E} - \tilde{\eta})$ is entirely positive. A sufficient condition for this is that the row sums of $\tilde{\eta}$ are strictly lower than 1.

This condition gives us the means to determine for a snapshot if a power allocation exists such that the E_b/N_0-requirements of all mobiles are met. If there is such a solution the NodeB's total transmit powers follows from Eq. (6) and the power allocated to each mobile from Eq. (2). By generating a series of system snapshots we obtain the moments or the distribution of the transmit powers under the condition that a reasonable power allocation exists. The advantage of the Monte Carlo simulation is that we can easily consider different service combinations, spatial processes, slow fading and imperfections of power control. However, the more stochastic values we include the more snapshots we require to obtain statistically relevant results. This makes the Monte Carlo simulation very time consuming. In order to obtain a bit less accurate results in a considerably shorter time we present an analytical model that approximates the NodeBs' transmit powers.

4 Approximate Methods

The approach behind the approximate method is to analytically derive the first and second moment of the transmit powers of all NodeBs. Then, we obtain the distribution of the transmit power by assuming that the part for the dedicated channels roughly follows a lognormal distribution. In the following, the number of users, their location and service are stochastic values according to the spatial traffic distribution and the service mix. Hence, the transmit powers $\hat{S}$ and the cell loads $\eta_{x,y}$ are stochastic values, as well. The mean of the transmit power of NodeB x follows by solving Eq. (5) for $\hat{S}_x$ and computing the mean:

$$\begin{aligned} \mathrm{E}\left[\hat{S}_x\right] &= \mathrm{E}\left[\tfrac{\hat{S}_{x,C}}{1-\eta_{x,x}}\right] + \mathrm{E}\left[\tfrac{\eta_{x,0}}{1-\eta_{x,x}}\right] W \hat{N}_0 \\ &+ \textstyle\sum_{y \neq x} \mathrm{E}\left[\hat{S}_y \tfrac{\eta_{x,y}}{1-\eta_{x,x}}\right] \end{aligned} \tag{7}$$

The mobile's locations are assumed to be independent and identically distributed such that $\mathrm{E}[\delta_x] = \mathrm{E}[\delta_{x,k}] = \mathrm{E}\left[1/\hat{d}_{x,k}\right]$ and $\mathrm{E}[\Delta_{x,y}] = \mathrm{E}\left[\hat{d}_{y,k}/\hat{d}_{x,k}\right]$. So we can separate the stochastic influence of the number of users and their service from that of the users' locations.

$$\mathrm{E}[\eta_{x,y}] = \begin{cases} \mathrm{E}\left[\sum_k \omega_k \frac{1}{\hat{d}_{x,k}}\right] = \mathrm{E}[\eta_x] \cdot \mathrm{E}[\delta_x] & \text{if } y = 0 \\ \mathrm{E}\left[\sum_k \omega_k \frac{\hat{d}_{y,k}}{\hat{d}_{x,k}}\right] = \mathrm{E}[\eta_x] \cdot \mathrm{E}[\Delta_{x,y}] & \text{if } y \neq x \\ \mathrm{E}\left[\sum_k \omega_k \alpha\right] = \mathrm{E}[\eta_x] \cdot \alpha & \text{if } y = x \end{cases} \tag{8}$$

Assuming independence of all random variables we obtain the expectation of the transmit power:

$$\begin{aligned} \mathrm{E}\left[\hat{S}_x\right] = \hat{S}_{x,C}\mathrm{E}\left[\tfrac{1}{1-\alpha\eta_x}\right] + W\hat{N}_0\mathrm{E}[\delta_x]\,\mathrm{E}\left[\tfrac{\eta_x}{1-\alpha\eta_x}\right] \\ + \textstyle\sum_{y\neq x} \mathrm{E}\left[\hat{S}_y\right]\mathrm{E}\left[\tfrac{\eta_x}{1-\alpha\eta_x}\right]\mathrm{E}[\Delta_{x,y}] \end{aligned} \tag{9}$$

Note that we neglect the dependence of $\hat{S}_y$ and the position of the mobiles in cell x and also the dependence between $\hat{S}_y$ and the load of cell x. In the latter point the "direct" method and the "state" method differ. In the "direct" method the load η_x remains a stochastic variable, we formulate a matrix equation and solve it directly:

$$\mathrm{E}\left[\bar{S}\right] = \bar{F}_1 + \tilde{G}_1\mathrm{E}\left[\bar{S}\right] \Rightarrow \mathrm{E}\left[\bar{S}\right] = \left(\tilde{E} - \tilde{G}_1\right)^{-1}\bar{F}_1 \tag{10}$$

with

$$F_1[z] = \hat{S}_{z,C}\mathrm{E}\left[\frac{1}{1-\alpha\eta_z}\right] + \mathrm{E}\left[\frac{\eta_z}{1-\alpha\eta_z}\right] W\hat{N}_0\mathrm{E}[\delta_z]$$

and

$$G_1[z_1, z_2] = \begin{cases} \mathrm{E}\left[\frac{\eta_{z_1}}{1-\alpha\eta_{z_1}}\right]\mathrm{E}[\Delta_{z_1,z_2}] & \text{if } z_1 \neq z_2 \\ 0 & \text{if } z_1 = z_2 \end{cases}$$

In the "state" method we consider the NodeBs separately. Let x be the NodeB in focus. Then we "make the load η_x deterministic" by computing the mean transmit powers under the condition that $\bar{n} = (n_1, ..., n_S)$ mobiles belong to NodeB x. Hence, we replace in Eq. (10) the stochastic variable η_x by the deterministic value $\eta(\bar{n}) = \sum_{s=1}^{S} n_s\omega_s$. The loads of the other NodeBs remain stochastic. This reduces the effect of the unconsidered correlations inherent in the matrix equation. In Section 5 we will see that this method has only a slight effect on the approximation of the mean transmit powers but considerably improves the standard deviations. However, the computational effort increases as we have to run through the state space of each NodeB twice.

First, for each NodeB x we compute the mean values required in Eq. (10) depending on the traffic intensity $a_{x,s}$ of service s at NodeB x

$$\begin{aligned} \mathrm{E}\left[\tfrac{1}{1-\alpha\eta_z}\right] &= \textstyle\sum_{\bar{n}|\eta(\bar{n})<1} p(\bar{n})\tfrac{1}{1-\alpha\eta(\bar{n})} \\ \mathrm{E}\left[\tfrac{\eta_z}{1-\alpha\eta_z}\right] &= \textstyle\sum_{\bar{n}|\eta(\bar{n})<1} p(\bar{n})\tfrac{\eta(\bar{n})}{1-\alpha\eta(\bar{n})} \end{aligned} . \tag{11}$$

The probability that $\bar{n}$ mobiles are active is

$$p(\bar{n}) = \prod_{s=1}^{S} \frac{a_{x,s}^{n_s}}{n_s!}. \tag{12}$$

The traffic intensities $a_{x,s}$ and the mean values of δ_x and Δ_x depend on the spatial distribution of the users and are computed separately by summing over all squares in $\mathcal{F}$. The propagation gain from a NodeB x to square f is $\hat{d}_{x,f}$. The probability that NodeB x covers point f, or short $f \in x$, is

$$p(f \in x) = P\left(\hat{d}_{x,f} = \min y \hat{d}_{y,f}\right). \tag{13}$$

The traffic intensity of service s at NodeB x is then

$$a_{x,s} = p_s \cdot \sum_{f \in \mathcal{F}} a_f \cdot p(f \in x) \tag{14}$$

and the means of δ_x and $\Delta_{x,y}$ are

$$\begin{aligned} \mathrm{E}\left[\delta_x\right] &= \textstyle\sum_{f \in F} \frac{a_f \cdot p(f \in x)}{\sum_{s=1}^{S} a_{x,s}} \mathrm{E}\left[\frac{1}{\hat{d}_{x,f}} \middle| f \in x\right] \\ \mathrm{E}\left[\Delta_{x,y}\right] &= \textstyle\sum_{f \in F} \frac{a_f \cdot p(f \in x)}{\sum_{s=1}^{S} a_{x,s}} \mathrm{E}\left[\frac{\hat{d}_{y,f}}{\hat{d}_{x,f}} \middle| f \in x\right] \end{aligned}. \tag{15}$$

These values are sufficient to obtain the mean values for the "direct" method. With the "state" method for each NodeB x we run over all states $\bar{n}$ again and obtain the conditioned mean transmit powers $\mathrm{E}\left[\hat{S}_x | \bar{n}\right]$. The overall mean transmit power follows by summing over all states $\bar{n}$:

$$\mathrm{E}\left[\hat{S}_x\right] = \textstyle\sum_{\bar{n} | \eta(\bar{n}) < 1} p(\bar{n}) \mathrm{E}\left[\hat{S}_x | \bar{n}\right] \tag{16}$$

Let us now consider the second moment of the Node-B transmit power. The square of Eq. (5) solved for $\hat{S}_x$ yields

$$\hat{S}_x^2 = \left(\frac{1}{1 - \alpha \eta_x}\right)^2 \left(\hat{S}_{x,C} + \eta_{x,0} W \hat{N}_0 + \sum_{y} \hat{S}_y \eta_{x,y}\right)^2. \tag{17}$$

Taking the mean of this equation and assuming the occurring random variables to be independent we obtain

$$\begin{aligned} \mathrm{E}\left[\hat{S}_x^2\right] = & \hat{S}_C^2 \mathrm{E}\left[\frac{1}{(1-\alpha\eta_x)^2}\right] \\ & + 2\hat{S}_C W \hat{N}_0 \mathrm{E}\left[\delta_x\right] \mathrm{E}\left[\frac{\eta_x}{(1-\alpha\eta_x)^2}\right] \\ & + 2\hat{S}_C \mathrm{E}\left[\frac{\eta_x}{(1-\alpha\eta_x)^2}\right] \sum_{y \neq x} \mathrm{E}\left[\hat{S}_y\right] \mathrm{E}\left[\Delta_{x,y}\right] \\ & + (W\hat{N}_0)^2 \mathrm{E}\left[\frac{\eta_{x,0}^2}{(1-\alpha\eta_x)^2}\right] \\ & + W\hat{N}_0 \sum_{y \neq x} \mathrm{E}\left[\hat{S}_y\right] \mathrm{E}\left[\frac{\eta_{x,0}\eta_{x,y}}{(1-\alpha\eta_x)^2}\right] \\ & + \sum_{y_1 \neq x} \sum_{y_2 \neq x} \mathrm{E}\left[\hat{S}_{y_1}\hat{S}_{y_2}\right] \mathrm{E}\left[\frac{\eta_{x,y_1}\eta_{x,y_2}}{(1-\alpha\eta_x)^2}\right] \end{aligned}. \tag{18}$$

In the last line the second moments of the other Node-Bs y occur if $y = y_1 = y_2$. Assuming independence of $\hat{S}_{y_1}$ and $\hat{S}_{y_2}$ for $y_1 \neq y_2$ we can write this equation in the following form:

$$\mathrm{E}\left[\hat{S}_x^2\right] = \bar{F}_2[x] + \sum_{y \neq x} \tilde{G}_2[x,y]\mathrm{E}\left[\hat{S}_y^2\right] \tag{19}$$

We formulate these equations for all Node-Bs in a matrix equation and solve for the vector $\mathrm{E}\left[\bar{S}^2\right]$ containing the second moments of the Node-Bs' transmit powers

$$\mathrm{E}\left[\bar{S}^2\right] = \bar{F}_2 + \tilde{G}_2\mathrm{E}\left[\bar{S}^2\right] \Rightarrow \mathrm{E}\left[\bar{S}^2\right] = \left(\tilde{E} - \tilde{G}_2\right)^{-1}\bar{F}_2 \tag{20}$$

Note that for the calculation of $\bar{F}_2[x]$ the mean transmit powers are required. It is defined as

$$\begin{aligned}
F_2[x] =& \hat{S}_C^2\mathrm{E}\left[\frac{1}{(1-\alpha\eta_x)^2}\right] \\
&+ 2\hat{S}_C W\hat{N}_0\mathrm{E}\left[\delta_x\right]\mathrm{E}\left[\frac{\eta_x}{(1-\alpha\eta_x)^2}\right] \\
&+ 2\hat{S}_C\mathrm{E}\left[\frac{\eta_x}{(1-\alpha\eta_x)^2}\right]\sum_{y\in\mathcal{B}\setminus x}\mathrm{E}\left[\hat{S}_y\right]\mathrm{E}\left[\Delta_{x,y}\right] \\
&+ (W\hat{N}_0)^2\mathrm{E}\left[\frac{{\eta_{x,0}}^2}{(1-\alpha\eta_x)^2}\right]L4 \\
&+ W\hat{N}_0\sum_{y\in\mathcal{B}\setminus x}\mathrm{E}\left[\hat{S}_y\right]\mathrm{E}\left[\frac{\eta_{x,0}\eta_{x,y}}{(1-\alpha\eta_x)^2}\right] \\
&+ \sum\sum_{y_1\neq y_2\in\mathcal{B}\setminus x}\mathrm{E}\left[\hat{S}_{y_1}\right]\mathrm{E}\left[\hat{S}_{y_2}\right]\mathrm{E}\left[\frac{\eta_{x,y_1}\eta_{x,y_2}}{(1-\alpha\eta_x)^2}\right]
\end{aligned} \tag{21}$$

and the matrix entry $G_2[x,y]$ is

$$G_2[x,y] = \begin{cases} 0, & \text{if } x = y \\ \mathrm{E}\left[\frac{\eta_{x,y^2}}{(1-\eta_x)^2}\right], & \text{if } x \neq y \end{cases}. \tag{22}$$

This leaves computing the expectations occurring in the definition of $\bar{F}$ and $\tilde{G}$. They are all computed according to the theorem of total probability:

$$\begin{aligned}
\mathrm{E}\left[\frac{1}{(1-\alpha\eta_x)^2}\right] &= \sum_{\bar{n}|\eta(\bar{n})<1} p(\bar{n})\frac{1}{(1-\alpha\eta(\bar{n}))^2} \\
\mathrm{E}\left[\frac{\eta_x}{(1-\alpha\eta_x)^2}\right] &= \sum_{\bar{n}|\eta(\bar{n})<1} p(\bar{n})\frac{\eta(\bar{n})}{(1-\alpha\eta(\bar{n}))^2} \\
\mathrm{E}\left[\frac{{\eta_{x,0}}^2}{(1-\alpha\eta_x)^2}\right] &= \sum_{\bar{n}|\eta(\bar{n})<1} p(\bar{n})\frac{\mathrm{E}\left[\eta_{x,0}(\bar{n})^2\right]}{(1-\alpha\eta(\bar{n}))^2} \\
\mathrm{E}\left[\frac{\eta_{x,0}\eta_{x,y}}{(1-\alpha\eta_x)^2}\right] &= \sum_{\bar{n}|\eta(\bar{n})<1} p(\bar{n})\frac{\mathrm{E}\left[\eta_{x,0}(\bar{n})\eta_{x,y}(\bar{n})\right]}{(1-\alpha\eta(\bar{n}))^2}. \qquad (23)
\end{aligned}$$

The probability $p(\bar{n})$ of a state $\bar{n}$ is already given in Eq. (12) so we show how to determine the expectation of the respective random variable for a state if it is not obvious:

$$\begin{aligned}
\mathrm{E}\left[\eta_{x,0}(\bar{n})^2\right] &= \mathrm{E}\left[\sum_{i\in x}\sum_{j\in x}\omega_i\omega_j\delta_{i,x}\delta_{j,x}\right] \\
&= \sum_{i\in x}\sum_{j\in x}\omega_i\omega_j\mathrm{E}\left[\delta_x\right]^2 - \underbrace{\left(\sum_{i\in x}\omega_i^2\right)}_{=\eta_x^2(\bar{n})}\mathrm{E}\left[\delta_x\right]^2 + \underbrace{\left(\sum_{i\in x}\omega_i^2\right)}_{=\eta_x^2(\bar{n})}\mathrm{E}\left[\delta_x^2\right] \\
&= \eta_x(\bar{n})\mathrm{E}\left[\delta_x\right]^2 + \eta_x^2(\bar{n})\mathrm{VAR}\left[\delta_x\right] \qquad (24)
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}\left[\eta_{x,0}(\bar{n})\eta_{x,y}(\bar{n})\right] &= \mathrm{E}\left[\sum_{i\in x}\sum_{j\in x}\omega_i\omega_j\delta_{i,x}\Delta_{j,x,y}\right] \\
&= \sum_{i\in x}\sum_{j\in x}\omega_i\omega_j\mathrm{E}\left[\delta_x\right]\mathrm{E}\left[\Delta_{x,y}\right] \\
&\quad - \sum_{i\in x}\omega_i^2\mathrm{E}\left[\delta_x\right]\mathrm{E}\left[\Delta_{x,y}\right] \\
&\quad + \sum_{i\in x}\omega_i^2\mathrm{E}\left[\delta_x\Delta_{x,y}\right] \\
&= \eta_x(\bar{n})\mathrm{E}\left[\delta_x\right]\mathrm{E}\left[\Delta_{x,y}\right] + \eta_x^2(\bar{n})\mathrm{COV}\left[\delta_x,\Delta_{x,y}\right] \qquad (25)
\end{aligned}$$

$$
\begin{aligned}
\mathrm{E}\left[\eta_{x,y_1}(\bar{n})\eta_{x,y_2}(\bar{n})\right] &= \mathrm{E}\left[\sum_{i\in x}\sum_{j\in x}\omega_i\omega_j\Delta_{i,x,y_1}\Delta_{j,x,y_2}\right] \\
&= \sum_{i\in x}\sum_{j\in x}\omega_i\omega_j\mathrm{E}\left[\Delta_{x,y_1}\right]\mathrm{E}\left[\Delta_{x,y_2}\right] \\
&\quad - \sum_{i\in x}\omega_i^2\mathrm{E}\left[\Delta_{x,y_1}\right]\mathrm{E}\left[\Delta_{x,y_2}\right] \\
&\quad + \sum_{i\in x}\omega_i^2\mathrm{E}\left[\Delta_{x,y_1}\Delta_{x,y_2}\right] \\
&= \eta_x(\bar{n})\mathrm{E}\left[\Delta_{x,y_1}\right]\mathrm{E}\left[\Delta_{x,y_2}\right] + \eta_x^2(\bar{n})\mathrm{COV}\left[\delta_x, \Delta_{x,y}\right] \qquad (26)
\end{aligned}
$$

The variances and covariances $\mathrm{VAR}[\delta_x]$, $\mathrm{COV}[\delta_x\Delta_{x,y}]$, and $\mathrm{COV}[\delta_x\Delta_{x,y}]$ are computed analog to Eq. (15) by integrating over all area elements.

The "state" method can also be applied for the second moment; in fact the computation of the conditional first and second moment requires only one additional run through the state space of each NodeB.

The main question for network planning is whether the networks carries the traffic or not. Let $p_{x,max}$ be the probability that the transmit power $\hat{S}_x$ exceeds the maximum allowed transmit power $\hat{S}_{x,max}$. Then, the network carries the traffic if for all NodeBs this probability stays below a certain threshold. The probability $p_{x,max}$ follows from the mean and variance by assuming that the variable part of the transmit power is approximately lognormal. Let μ_x and σ_x^2 be the parameters of the lognormal distribution with mean $(\mathrm{E}[\hat{S}_x] - \hat{S}_C)$ and variance $(\mathrm{E}[\hat{S}_x^2] - \mathrm{E}[\hat{S}_x]^2)$. Then the probability $p_{x,max}$ is

$$
p_{x,max} = 1 - LN_{\mu_x,\sigma_x^2}\left(\hat{S}_{x,max} - \hat{S}_C\right), \qquad (27)
$$

where LN_{μ,σ^2} denotes the CDF of the lognormal distribution.

5 Numerical Results

In this section we consider two scenarios in order to compare the mean and standard deviation of the NodeBs transmit powers computed by the different methods. Scenario 1 considers a network with 19 NodeBs in a hexagonal grid and homogeneous traffic as shown in Figure 1. The network of Scenario 2 consists of 22 NodeBs that are arranged arbitrarily. The spatial traffic distribution is also heterogeneous. Figure 2(a) shows the NodeB positions and the spatial traffic distribution. The darker the square, the higher is the amount of traffic.

In both scenarios we consider the following two services:

service	bit-rate	E_b/N_0-target	probability
1	12.2kbps	5.5dB	0.5
2	64kbps	4 dB	0.5

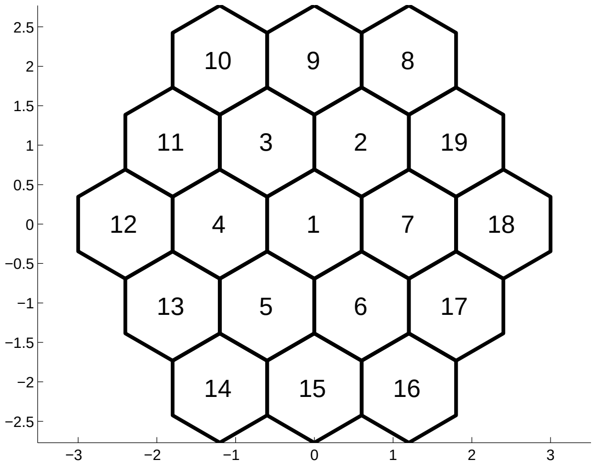


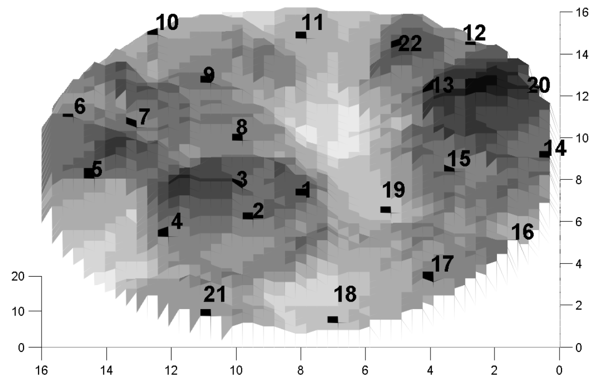
Figure 1: Hexagonal NodeB layout of Scenario 1

The traffic intensity of both scenarios is chosen such that there is a mean of 12 users per NodeB and service. In Scenario 1 the resulting traffic intensities are $a_{x,s} = 12$ for all NodeBs x and services s. The resulting traffic intensities for Scenario 2 are shown in Figure 2(b). We consider the simple deterministic propagation model from [9]:

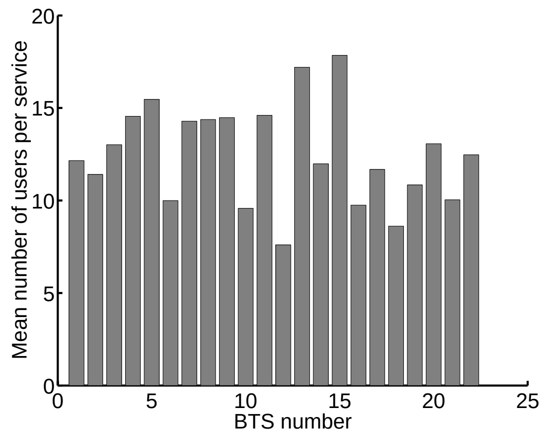
$$d_{x,k} = -128.1 - 37.6 \log_{10}(\text{dist}(x, k)) \tag{28}$$

The constant power for the common channels is set to $\hat{S}_{x,C} = 2000mW$ for all NodeBs.

Figure 3(a) shows the mean transmit powers for Scenario 1. The solid line marks the results from the Monte Carlo Simulation with 50000 snapshots such that the 90%-confidence intervals are hardly discernable. The triangles mark the curve of the “direct” method and the circle the one of the “state” method. Both approximate curves agree very well with the Monte Carlo simulation. The NodeB numbers on the x-axis correspond to the numbers given in Figure 1. The central NodeB requires the highest transmit power as its mobiles experience the most interference from the surrounding NodeBs. The NodeBs in the first tier need about 100mW less power in average. The gap to the second tier is much larger as those NodeBs have only three neighbored NodeBs. The NodeBs in the second tier with odd numbers are nearer to the center than those with even numbers such that they require about 200mW more power in average. The figure also shows that we have to consider at least two tiers in order to get accurate results for the central NodeB. Figure 3(b) shows the corresponding curves for the standard deviations. The approximate methods deviate from the Monte Carlo simulation for the central NodeB and the first tier. The results for the second tier match quite well. Here, the improvement by the “state” method becomes evident as the difference to the Monte Carlo simulation is roughly reduced by half. Another interesting feature which we can observe when comparing Figures 3(a) and 3(b) is that the variation coefficient of the power for dedicated channels, i.e. $(\hat{S}_x - \hat{S}_{x,C})$, is about 0.35 for all NodeBs.



(a) Spatial user distribution

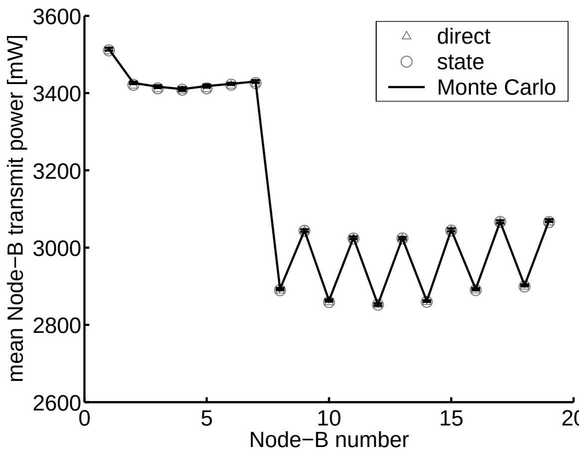


(b) Load distribution among base stations

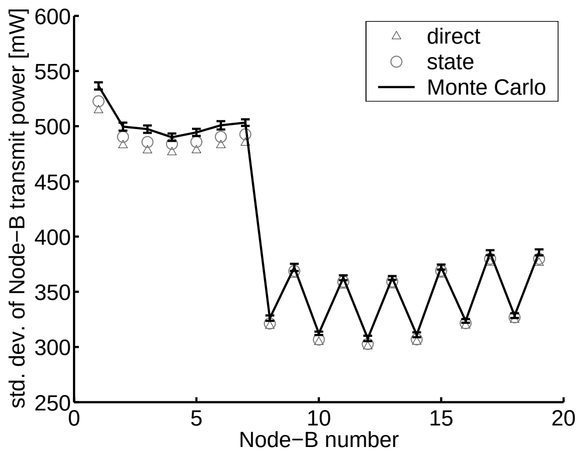
Figure 2: Network of Scenario 2

Let us now consider the more challenging case of Scenario 2 with the heterogeneous traffic. Figure 4(a) shows the mean transmit powers and Figure 4(b) the corresponding standard deviations. The most obvious thing to observe is that the mean transmit powers are more different and much higher though the mean traffic density per NodeB is the same. The reasons for this are: First, that the mean covered area per NodeB is larger in Scenario 2. Second, that the NodeB arrangement is not optimal such that larger distances from NodeB to mobile occur. And third, that a NodeB with higher traffic density requires considerably more power and therefore forces the power of other NodeBs up, as well. The approximate results match again well for the mean transmit powers and the standard deviations are again underestimated, in particular for NodeBs with large power. The "state" method is again better than the "direct" method, however, the gap to the Monte Carlo simulation is here larger as the values itself are larger, too. Furthermore, it's interesting to observe that NodeB 15 with the highest traffic intensity does not require the most transmit power. Instead, the power requirement of NodeB 8 is the largest.

Finally, the question whether the network carries the traffic is answered for Scenario 2. Fig. 5(a) shows the probabilities that the transmit power exceeds a maximum of $\hat{S}_{x,max} = 10W$. Most of the NodeBs keep this probability clearly below one percent. NodeBs 13 and 15 experience an outage probability of about one percent and the highest outage has NodeB 8 with 2.6 percent. Figure 5(b) shows the outage for NodeB 8 with an average load of 20 to 30 users per NodeB. The outage probability increases from almost zero up to over 30%. In both figures the approximations underestimate the outage probabilities as expected from the too low standard deviations. The underestimation becomes larger for NodeBs with larger transmit powers. However, the approximate results match well enough to decide whether a network is capable to carry the traffic or not.



(a) mean



(b) standard deviation

Figure 3: Statistical values of the transmit powers in Scenario 1

6 Conclusion

At the beginning of the paper stood the question if a UMTS network is able to carry the offered traffic. A scenario was defined to be a network with a spatial traffic distribution and a service mix. A Monte Carlo simulation technique and two approximate methods, "direct" and "state", were formulated to compute the outage probabilities for the Node-Bs of a scenario. The approximate methods show good results that slightly underestimate the results from the Monte Carlo simulation. The reason for this is that the correlations between the Node-Bs' transmit powers are not completely considered. The underestimation is larger for Node-Bs with higher transmit power. Nevertheless, the method provides an efficient way for a network operator to evaluate for a general scenario whether the network is well-designed or not.

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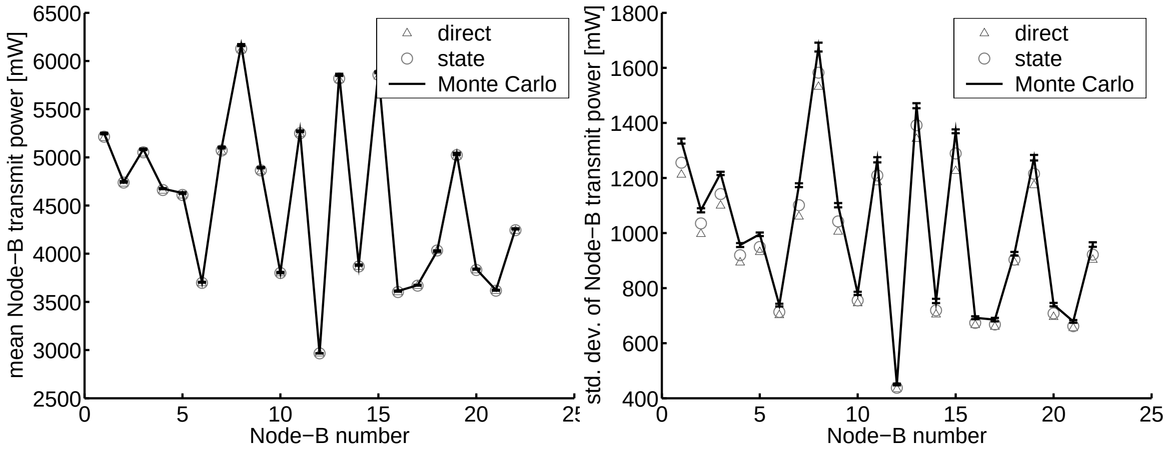

(a) mean (b) standard deviation

Figure 4: Statistical values of the transmit powers in Scenario 2

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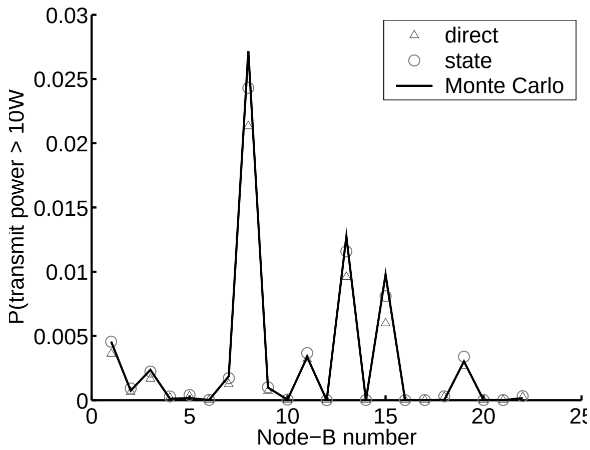
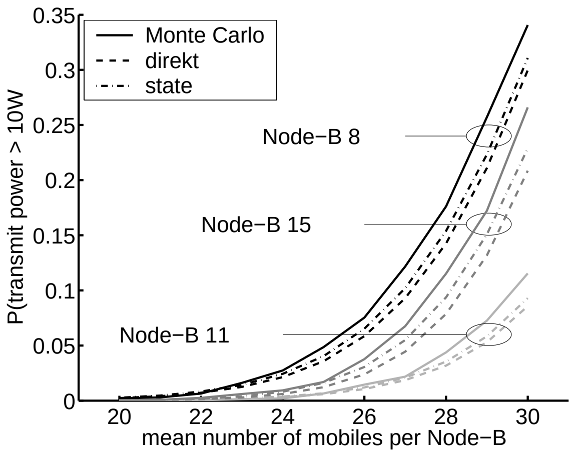


(a) Standard Scenario 2



(b) NodeB 8 of Scenario 2 with varying load

Figure 5: Outage probability in Scenario 2